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ELECTRODYNAMICS, AND THE MUTUAL ANNIHILATION OF POSITIVE AND NEGATIVE ELECTRICITY.

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ABSTRACT.

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The mutual annihilation of positive and negative electricity with an accompanying radiation of the equivalent electromagnetic energy has been invoked to account for stellar heat. Electrodynamics, without further generalization, has no way of admitting such annihilation accompanied by radiation. The generalizations necessary to provide logical meaning to the process are developed and discussed. In particular, it becomes necessary to extend the law of motion of electricity applicable to an electron as a whole, so as to apply also to that which remains of the electron when part of it is destroyed.

INTRODUCTION.

The mutual annihilation of positive and negative electricity has assumed a position of importance in relation to the origin of stellar radiant energy. It is probable that the true story of such annihilation cannot be told without an appeal to principles, such as those of wave-mechanics, more widely inclusive than those of electrodynamics, even when modified within reason. However, it is of some interest to enquire as to what classical electrodynamics really has to say upon the matter, and to investigate what generalizations, if any, are necessary, in order to give meaning to the processes involved.

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GENERAL STATEMENT OF ELECTRODYNAMIC PRINCIPLES.

It will be recalled that the equations of classical electrodynamics are:

$$\rho \frac{\mathbf{u}}{c} = -\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \text{curl } \mathbf{H}, \tag{1}$$

$$\rho = \text{div. } \mathbf{E}, \tag{2}$$

$$0 = \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} + \text{curl } \mathbf{E}. \tag{3}$$

$$0 = \text{div. } \mathbf{H} \tag{4}$$

supplemented by a "force equation" giving the law according to which electrons [1] move under the influence of the fields $\mathbf{E}$ and $\mathbf{H}$. This force equation determines the motion of the electron to be such as to provide for

$$\int\int\int \mathbf{E}'\rho' d\tau' = 0, \tag{5}$$

where the integral is evaluated under the following conditions: It is supposed that there exists a set of axes in which coordinate values are denoted by x', y', z', t' and in which the electron is a sphere all of whose parts have zero velocity. If we denote this system of coördinates by S', then $\mathbf{E}'$, ρ', and $d\tau'$ are the electric field, charge density and element of volume, evaluated in S'. The assumptions that the electron is a sphere and that the charge is concentrated on the surface of the sphere, while customary, are not necessary in order that equation (5) may serve as a basis on which a logical theory may be built. All that is necessary is that there shall exist a system S' in which all the parts of the electron have zero velocity. The assumption of a spherical surface distribution of charge is, of course, necessary to lead to the customary expression for the electromagnetic mass, although a symmetrical volume distribution leads to the same value except for a constant factor. The integration in (5) is, of course, carried out at constant t'.

If

$$k \equiv (1 - q^2/c^2)^{-1/2},$$

[1] We use the term "electrons" to include electrons of either sign.

where q is the resultant velocity of the element of charge at the point considered, it is well known that $k(\mathbf{E} + [\mathbf{uH}]/c)$, $ik(\mathbf{E}\cdot\mathbf{u})/c$ constitute a 4-vector, and it is easy, therefore, to show that (5) leads to [2]

$$\iiint_{r = \text{const.}} \left(\mathbf{E} + \frac{[\mathbf{uH}]}{c} \right) \rho d\tau = 0 \tag{6}$$

and to

$$\iiint_{r = \text{const.}} \left(\rho\mathbf{u}\cdot\overline{\mathbf{E} + \frac{[\mathbf{uH}]}{c}} \right) d\tau = 0 \tag{7}$$

or its equivalent

$$\iiint_{r = \text{const.}} (\rho\mathbf{u}\cdot\mathbf{E}) d\tau = 0. \tag{8}$$

The fields $\mathbf{E}$ and $\mathbf{H}$ may be split up into two parts. First there is a part $\mathbf{E}_i$, $\mathbf{H}_i$, calculable, from the charge whose motion is under consideration, through the agency of the well-known solutions of (1)–(4),

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{U}}{\partial t} - \text{grad } \varphi, \tag{9}$$

$$\mathbf{H} = \text{curl } \mathbf{U}, \tag{10}$$

where

$$4\pi c\mathbf{U} = \iiint \frac{[\rho\mathbf{u}]}{r} d\tau; \qquad 4\pi\varphi = \iiint \frac{[\rho]}{r} d\tau, \tag{11}$$

subject to [3]

$$\text{div. } \mathbf{U} = -\frac{1}{c}\frac{\partial \varphi}{\partial t}, \tag{12}$$

where the square brackets here imply that the integrals are to be calculated in the usual retarded sense, and where ρ and $\mathbf{u}$ refer to the charge whose motion is considered. Then there is another part $\mathbf{E}_0$, $\mathbf{H}_0$, constituting the "external" field as

[2] These results are well known. The intermediate step in their derivation will be found in the article immediately preceding the present one in the JOURNAL, "Classical Electrodynamics and the Conservation of Energy," JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 212, p. 563, Nov. 1931.

[3] This condition merely preserves the equation of continuity for the electricity.

regards this charge, and calculable in similar manner from the other charges of the system, or possibly in part as a solution of equations (1)–(4) with ρ zero. Equation (6) may then be written as

$$\iiint_{t' = \text{const.}} \left(\mathbf{E}_0 + \frac{[\mathbf{u}\mathbf{H}_0]}{c} \right) \rho d\tau = - \iiint_{t' = \text{const.}} \left(\mathbf{E}_i + \frac{[\mathbf{u}\mathbf{H}_i]}{c} \right) \rho d\tau. \quad (13)$$

The integral on the right may be evaluated in terms of the motion of the charge concerned.[4] It consists of a part involving time derivatives no higher than the acceleration, and a part-F involving higher derivatives and associated with radiation and the resulting damping forces on the charge. In the case of a spherical shell of charge e of radius a, we have, of course, the well-known expression

$$\iiint_{t' = \text{const.}} \left(\mathbf{E}_0 + \frac{[\mathbf{u}\mathbf{H}_0]}{c} \right) \rho d\tau = \frac{e^2}{6\pi a c^2} \frac{d}{dt} \frac{\mathbf{w}}{\left(1 - \frac{q^2}{c^2} \right)^{1/2}} + \mathbf{F}, \quad (14)$$

where $\mathbf{w}$ is the velocity of the center of the electron.

It is a well-known algebraical consequence of equations (1)–(4) that

$$\iiint \left(\rho \mathbf{u} \cdot \mathbf{E} + \frac{\overline{[\mathbf{u}\mathbf{H}]}}{c} \right) d\tau = - \frac{\partial}{\partial t} \iiint \left(\frac{\mathbf{E}^2 + \mathbf{H}^2}{2} \right) d\tau - c \iint [\mathbf{E}\mathbf{H}]_n dS. \quad (15)$$

The result is true as regards the whole field $\mathbf{E}$, $\mathbf{H}$, or any part thereof which, in conjunction with $\rho\mathbf{u}$ and ρ forms a solution of (1)–(4). The integrals are, of course, evaluated at t = constant.

Now in a previous paper in a former issue of the Journal,[5] considerable discussion has been given as to the significance of

[4] It may most easily be evaluated by equating it to the corresponding expression in the system S' and then evaluating it in this system.

[5] JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 212, pp. 563–576, 1931.

equation (15). It will suffice to recapitulate here some of the main conclusions. The fundamental equation of motion does not provide, as a consequence of (15), for the relation

$$\mathbf{v}\int\int\int\left(\mathbf{E}_0+\frac{[\mathbf{u}\mathbf{H}_0]}{c}\right)\rho d\tau=\frac{\partial}{\partial t}\int\int\int\left(\frac{\mathbf{E}_i^2+\mathbf{H}_i^2}{c}\right)+c\int\int[\mathbf{E}_i\mathbf{H}_i]_n dS, \quad (16)$$

between what is customarily regarded as the rate of doing work by the external field on the one hand, and the rate of increase of energy of the field of the electron within a boundary plus the flux through that boundary on the other hand. In equation (16) there must be included another term which is of the same order of magnitude as the right-hand side of (16) itself. The point involved is the old story of the Lorentz electron and the conservation of energy. The trouble is not due to the evaluation of the integrals in (13) at t' = constant rather than at t = constant. By a modification of the equation of motion (13) either by the inclusion of a suitable factor different from unity, on the right-hand side, or by the addition of a suitably chosen function of the velocity and acceleration of the charge concerned, it is possible to provide for the truth of equation (16) as it stands, except for certain corrections which now, however, are of a different order of magnitude from the discrepancy which existed before. This modification moreover carries with it the conclusion that (7) or (8) holds with the integral evaluated at t = constant rather than at t' = constant. Under these conditions, the left-hand side of (15) is zero, and we have

$$\frac{\partial}{\partial t}\int\int\int\left(\frac{\mathbf{E}^2+\mathbf{H}^2}{2}\right)+c\int\int[\mathbf{E}\mathbf{H}]_n dS=0. \quad (17)$$

It is upon the equation of motion of the charge that the interpretation of (15) ultimately rests, and some such modification of the classical equation as is indicated in the paper referred to is necessary to provide for (16) and (17), for unless these equations are provided for (15) remains as an analytical truth devoid of much important significance. It is of importance to recognize that this matter is not one concerned

only with the application of the energy equation to the problem of annihilation of charge, which is the main object of this paper, but is one pertaining to every application of that equation.

Two possible equations of motion suitable for the desired end have been considered in the preceding paper already cited. They are

$$\int\int\int_{r' = \text{const.}} \left(\mathbf{E} + \frac{[\mathbf{uH}]}{c}\right) \rho d\tau = \frac{\alpha}{c^2}\frac{d}{dt}(k\mathbf{w}) \qquad (18)$$

and

$$\int\int\int_{r' = \text{const.}} \left(\mathbf{E}_0 + \frac{[\mathbf{uH}_0]}{c}\right) \rho d\tau = -(1+\lambda) \int\int\int_{r' = \text{const.}} \left(\mathbf{E}_i + \frac{[\mathbf{uH}_i]}{c}\right) \rho d\tau, \qquad (19)$$

where α and λ are constants calculable in terms of the charge distribution considered in general as arbitrary in a system of axes in respect to which it is at rest. Even these modifications of the equation of motion of a charge only serve the end in view to a certain degree of approximation, for the consideration of which reference must be made to the preceding article already cited.

INADEQUACY OF THE ELECTRODYNAMIC EQUATIONS FOR SUPERPOSED CHARGES.

Strictly speaking, equations (1)–(4) and equation (5) or the modification of it which we have referred to have no significance as applied to superposed charges. Thus, consider two spheres of electricity exactly alike in all respects except that the density is positive in one and negative in the other, and the velocity of one is oppositely directed to the velocity of the other at the instant of superposition. Under these conditions, ρ can have no meaning in (1)–(5), other than zero, and u has no way of expressing the two oppositely directed velocities in such a manner as to give what we know would be the contribution to the field resulting from the combined motions, of the superposed charges, in opposite directions. This defect is, of course, immediately remedied by concentrating attention

separately upon ρ_+ and ρ_-, with the corresponding $\mathbf{u}_+$ and $\mathbf{u}_-$. The corresponding fields $\mathbf{E}_+$, $\mathbf{H}_+$, and $\mathbf{E}_-$, $\mathbf{H}_-$ are to be considered as calculated from (9)–(12), in which $\mathbf{U}$ and φ are to be given "plus" subscripts and are calculable from ρ_+ and $\mathbf{u}_+$ when $\mathbf{E}_+$ and $\mathbf{H}_+$ are desired, and "minus" subscripts and calculate from ρ_- and $\mathbf{u}_-$ when $\mathbf{E}_-$ and $\mathbf{H}_-$ are desired. Defined in this way, $\mathbf{E}_+$, $\mathbf{H}_+$, and $\mathbf{E}_-$, $\mathbf{H}_-$, obey the equations [6]

$$\frac{\rho_\pm \mathbf{u}_\pm}{c} = -\frac{1}{c}\frac{\partial \mathbf{E}\pm}{\partial t} + \text{curl } \mathbf{H}\pm, \tag{20}$$

$$\rho_\pm = \text{div. } \mathbf{E}\pm, \tag{21}$$

$$0 = \frac{1}{c}\frac{\partial \mathbf{H}\pm}{\partial t} + \text{curl } \mathbf{E}\pm, \tag{22}$$

$$0 = \text{div. } \mathbf{H}\pm. \tag{23}$$

On subtracting the equations for negative charges from those for positive charges and defining $\mathbf{E}$ and $\mathbf{H}$ as

$$\mathbf{E} = \mathbf{E}_+ - \mathbf{E}_- + \mathbf{g}; \qquad \mathbf{H} = \mathbf{H}_+ - \mathbf{H}_- + \mathbf{h},$$

where $\mathbf{g}$ and $\mathbf{h}$ are, respectively, any electric and magnetic fields which form solutions of (1)–(4) with ρ zero, we have

$$\frac{1}{c}(\rho_+\mathbf{u}_+ - \rho_-\mathbf{u}_-) = -\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \text{curl } \mathbf{H}, \tag{24}$$

$$\rho_+ - \rho_- = \text{div. } \mathbf{E}, \tag{25}$$

$$0 = \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} + \text{curl } \mathbf{E}, \tag{26}$$

$$0 = \text{div. } \mathbf{H}. \tag{27}$$

These equations have meaning for superposed charges of positive and negative electricity. Moreover, the two sets of equations (20)–(23), through the equations of continuity

$$\frac{\partial \rho_+}{\partial t} + \text{div. } \rho_+\mathbf{u}_+ = 0; \qquad \frac{\partial \rho_-}{\partial t} + \text{div. } \rho_-\mathbf{u}_- = 0, \tag{28}$$

[6] When ρ and $\mathbf{u}$ occur without subscripts, they are supposed to carry these signs with them. Thus ρ is negative for a negative. When they carry a subscript, they will be regarded as positive, the sign being taken care of externally. Thus ρ_- is a positive quantity.

which they imply, conserve the positive and negative electricity separately. Even in the case of two superposed spheres, alike in all respects except in the oppositeness of their charge densities, so that $\rho_+ - \rho_-$ is zero everywhere, equations (20)–(23) are conscious of the separate existence of the superposed parts.

THE PSEUDO-STORY OF ANNIHILATION IN TERMS OF EQUATIONS (20)–(23), AND (24)–(27), WHICH LATTER ARE ESSENTIALLY OF THE CLASSICAL TYPE.

To illustrate the matter which we desire to discuss, let us consider two spheres of electricity equal in size and of equal density, one positive and the other negative, which, initially, are moving towards each other. As the spheres proceed to approach each other, each moves under the condition imposed by the appropriate force equation. We may think of each one as producing an electromagnetic field which acts as the "external" field which controls the other. Thus, for example, if (19) were taken as the fundamental form of the equation of motion, the motion of the negative sphere would be given by

$$\iiint_{t_-' = \text{const.}} \left(\mathbf{E}_+ + \frac{[\mathbf{u}_- \mathbf{H}_+]}{c} \right) \rho_- d\tau = -(1 + \lambda_-) \iiint_{t_-' = \text{const.}} \left(\mathbf{E}_- + \frac{[\mathbf{u}_- \mathbf{H}_-]}{c} \right) \rho_- d\tau \quad (29)$$

and the motion of the positive by

$$\iiint_{t_+' = \text{const.}} \left(\mathbf{E}_- + \frac{[\mathbf{u}_- \mathbf{H}_-]}{c} \right) \rho_+ d\tau = -(1 + \lambda_+) \iiint_{t_+' = \text{const.}} \left(\mathbf{E}_+ + \frac{[\mathbf{u}_+ \mathbf{H}_+]}{c} \right) \rho_+ d\tau. \quad (30)$$

Here t_-' refers to time as measured in a system in which the negative is at rest, and t_+' refers to time as measured in a system in which the positive is at rest. *The story continues to be told consistently in this form even at the stage where the spheres are passing through each other.* Physical intuition may demand some other forces as coming into play when the spheres penetrate, but physical intuition has no standing except in

so far as it is supported by the equations which one has decided to take as the basis of his calculations.

If in working out the right-hand side of equations (29) and (30) we neglect the terms corresponding to radiation retardation, the spheres would swing backwards and forwards through each other forever if they started from rest originally. Or, if they did not start from rest, they might, if the velocities were appropriate, pass through each other and then proceed on to infinity. With radiation taken into account, however, unless their initial velocities are too great, they will continue to swing backwards and forwards through each other with diminishing amplitude until they finally come to a permanently and completely superposed condition.

Let us consider some surface σ surrounding both spheres at the time when we first considered them. After they have been superposed for a sufficiently long time, we see from (24)–(27) that they will produce no field within σ. We shall prove that between these times there has been a radiation of energy, in the sense of a Poynting flux, through σ equal to the total amount of energy of the type $(\mathbf{E}^2 + \mathbf{H}^2)/2$ per unit of volume contained initially within the surface σ.

In the first place, it is easy to see that just as equations (1)–(4) lead, as an algebraical consequence to (15), so equations (24)–(27) lead directly to

$$\iiint (\{\rho_+\mathbf{u}_+ - \rho_-\mathbf{u}_-\}\cdot\mathbf{E})d\tau = -\frac{\partial}{\partial t}\iiint\left(\frac{\mathbf{E}^2+\mathbf{H}^2}{2}\right)d\tau - c\iint[\mathbf{EH}]_n dS \quad (31)$$

the integrals being in each case evaluated throughout the region bounded by σ. In fact, in the proof of (15) the first step consists in realizing that the left-hand side may be written as $\int\int\int(\rho\mathbf{u}\cdot\mathbf{E})d\tau$, and the next step consists in substituting for $\rho\mathbf{u}$, the value given by the right-hand side of (1). The first step in the proof of (31) consists in substituting for $(\rho_+\mathbf{u}_+ - \rho_-\mathbf{u}_-)$ from (24), exactly the same expression as was substituted for $\rho\mathbf{u}$ in the standard case. The remainders of the proofs are absolutely the same for each case. The integrals in (31) are, of course, evaluated at t = constant. Now

we have already discussed at length in the present paper and in the preceding paper the elements involved in insuring that the force equation shall be such as to provide for

$$\int\int\int (\rho \mathbf{u}\cdot\mathbf{E})d\tau = 0$$

with the integral evaluated at t = constant rather than at t' = constant as would be required by the classical theory. We shall not pursue this matter any further than to say that if it be denied that the integral be evaluated at t = constant, then not only will our attempts to provide a significance to the annihilation of charge with a corresponding radiation of energy be destroyed—at least as a "quantitative" fact, but the meaning of work and electromagnetic energy in ordinary problems of electrodynamics will be, at best, spoiled as regards useful significance. We shall assume that

$$\int\int\int_{t=\text{const.}} (\rho_+ \mathbf{u}_+\cdot\mathbf{E}) = 0, \tag{32}$$

$$\int\int\int_{t=\text{const.}} (\rho_- \mathbf{u}_-\cdot\mathbf{E}) = 0, \tag{33}$$

so that

$$\int\int\int (\{\rho_+ \mathbf{u}_+ - \rho_- \mathbf{u}_-\}\cdot\mathbf{E})d\tau = 0, \tag{34}$$

and from (31)

$$\frac{\partial}{\partial t}\int\int\int \left(\frac{\mathbf{E}^2 + \mathbf{H}^2}{2}\right) = -c\int\int [\mathbf{EH}]_n dS. \tag{35}$$

Integrating this result between the initial and final states corresponding to $t = t_1$ and $t = t_2$, respectively, we find

$$\left[\int\int\int \left(\frac{2}{\mathbf{E}^2 + \mathbf{H}^2}\right) d\tau\right]_{t_1}^{t_2} = \int_{t_1}^{t_2} dt \int\int c[\mathbf{EH}]_n dS. \tag{36}$$

Now, as already stated, the final state of our problem is one where the charges are superposed and there is no field whatever within σ. This state will certainly exist at all times later than r/c after complete superposition, where r is the greatest distance from any point on the superposed charges to the

boundary σ. Thus (36) gives

$$\iiint_{\text{at time } t_1} \left(\frac{\mathbf{E}^2 + \mathbf{H}^2}{2}\right) d\tau = \int_{t_1}^{t_2} dt \iint c[\mathbf{EH}]_n dS. \quad (37)$$

Or, stated in words, the superposition of the charges is accompanied by a total flux of energy, in the Poynting flux sense, through σ equal in amount to the total electromagnetic energy, in the sense $(\mathbf{E}^2 + \mathbf{H}^2)/2$ per unit of volume, contained initially within σ.

AN UNSATISFACTORY ELEMENT.

Although, in the foregoing example, the charges end by being superposed, they are not annihilated. Equations (28) provide for the permanence of existence of each of the superposed parts. The charges are "down" but not "out." Thus the application of an external field to the superposed pair would, through the application of the equation of motion (18) or (19) or whatever equations of this general type we may use, demand that they separate—partially if the external field is small and completely if it is large enough. It is important, in this connection, to dissect the pseudo-demands of intuition from the exact story of the equations which are taken as governing the course of events. Equations (28), which are the logical outcome of (20)–(23), provide for the permanence of the positive and negative charges, and the subsequent story of their separation by an electric field is simple and definite.

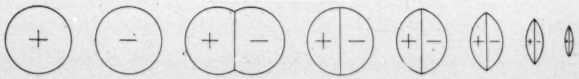
MODIFICATION OF CONCEPTS LEADING TO A TRUE MUTUAL ANNIHILATION OF POSITIVE AND NEGATIVE ELECTRICITY.

A true mutual annihilation of positive and negative electricity involves principles which may be illustrated by the following example: Let us commence by considering two spheres alike in all respects except in the oppositeness of the signs of their charge densities, and let them approach each other with equal velocities as shown in Fig. 1. If we suppose that each charge completely devours the other as they come together, the situation at successive stages in the process will be as depicted in the figure. Each sphere retains to the last the full value of its charge density at points which have not yet reached the other charge, but there is a steady mutual an-

nihilation at the boundary until finally, at the last stage, there is nothing left of either.

If the spheres are of unequal or possibly variable density and of unequal velocity, the boundary of annihilation does not stay fixed in position, but moves, at each point, in a direction

FIG. 1.



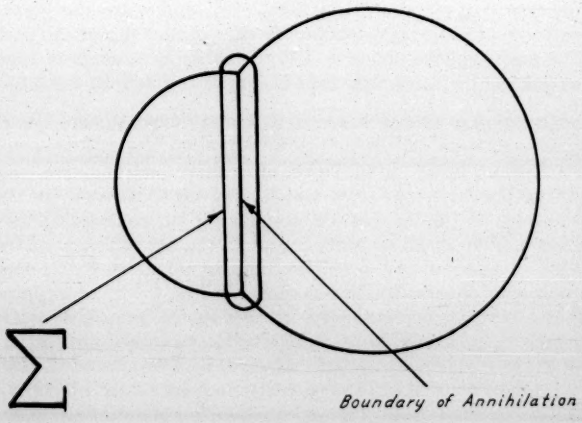
parallel to its normal with a velocity $\mathbf{w}$ such that, if $\mathbf{u}_+$ and $\mathbf{u}_-$ are the actual velocities of the positive and negative electricity, respectively, at the boundary,

$$\rho_+(\mathbf{u}_+ - \mathbf{w}) + \rho_-(\mathbf{u}_- - \mathbf{w}) = 0,$$

$$\mathbf{w} = \frac{\rho_+\mathbf{u}_+ - \rho_-\mathbf{u}_-}{\rho_+ + \rho_-}.$$

Let us surround the boundary of annihilation by a surface Σ, Fig. 2, which grows or diminishes and moves along with it.

FIG. 2.



At any point outside of Σ, equations (1)–(4) are valid, the derivatives of ρ and $\rho\mathbf{u}$ exist, and the equation of continuity

$$\frac{\partial \rho}{\partial t} + \text{div. } \rho\mathbf{u} = 0$$

holds. However, at a point on the boundary of annihilation itself, the derivatives of $\rho\mathbf{u}$ do not exist, so that the equation of continuity has no meaning as applied at such a point. However, the process of annihilation at the boundary does not deny the truth of the equation of continuity in any region where the equation itself has meaning. The annihilation takes place harmoniously, without any logical difficulties as regards the story told by equations (1)–(4). As a matter of fact, the vectors $\mathbf{E}$ and $\mathbf{H}$ are continuous even across the boundary itself, and their derivatives exist on the boundary.

It now remains to inquire as to the radiation of electromagnetic energy associated with this more drastic concept of annihilation of the charges.

As already remarked, it is an analytical consequence of (1)–(4) that

$$\iiint_{t=\text{const.}} (\rho\mathbf{u}\cdot\mathbf{E})d\tau = -\frac{\partial}{\partial t}\iiint_{t=\text{const.}} \left(\frac{\mathbf{E}^2+\mathbf{H}^2}{2}\right) d\tau - \iint_{t=\text{const.}} c[\mathbf{EH}]_n dS. \quad (38)$$

The volume integrals are evaluated throughout the region comprised between σ and Σ, where σ is a distant surface surrounding both charges and the surface integrals are evaluated over σ and Σ. In virtue of the continuity of the vectors $\mathbf{E}$ and $\mathbf{H}$ across the boundary of annihilation, the contributions to the surface integral over Σ by the closely abutting portions which compose it just cancel, so that we are left only with the surface integral over σ. Again, the continuity of $\mathbf{E}$ and $\mathbf{H}$ across the boundary of annihilation enables us to extend the volume integrals on the right-hand side of (38) right across the boundary, so that as regards the right-hand side we may think of the surface integral as being taken only over σ and the volume integral as taken throughout the region contained within σ.

We may now integrate both sides of (38) from t_1 to t_2 where t_2 corresponds to a time sufficiently long after the complete annihilation to insure that the field is everywhere zero within σ. If, and only if, the left-hand side gives zero value when integrated shall we arrive at the conclusion

$$\underset{\text{Evaluated at } t_1}{\iiint}\left(\frac{\mathbf{E}^2+\mathbf{H}^2}{2}\right)d\tau = -\int_{t_1}^{t_2} dt \iint c[\mathbf{EH}]_n dS, \quad (39)$$

which is the result enabling us to conclude that the complete annihilation of the charges is accompanied by a total time integral of Poynting flux through σ equal to the electromagnetic energy originally contained within σ.

The time integral of the left-hand side of (38) will be zero if,[7] and only if,

$$\underset{t=\text{const.}}{\iiint}(\rho_+\mathbf{u}_+\cdot\mathbf{E})d\tau - \underset{t=\text{const.}}{\iiint}(\rho_-\mathbf{u}_-\cdot\mathbf{E})d\tau = 0.$$

In other words, it is necessary that our equations of motion (18) or (19), or whatever the appropriate ones to provide for (32) and (33) may be, shall hold not merely for the whole spheres before the annihilation began, but it must also hold independently for each part of each sphere which remains as the annihilation proceeds. This does not mean that the law must necessarily hold for each element of volume, but it does mean that the remnant of each charge must, at each instant, constitute itself as a separate entity for which the law does hold, and must move at that instant in accordance with principles consistent with that law. Such constants as α and λ occurring in (18) and (19) will, of course, change with the amount of charge left at any instant.

The obedience of the charges to the foregoing criterion results in certain rather peculiar consequences. It will be recalled that the electromagnetic mass of a charge e, of radius a, is proportional to e^2/a, whether the charge is on the surface or is distributed throughout the volume, the factor of proportionality being, of course, different in the two cases. Expressed in the case of the density for a uniformly charged

[7] We discard as irrelevant cases of accidental cancellation of the time integral over different portions of the range t_1 to t_2 without the integral's being zero at each instant.

sphere, the mass is proportional to $\rho^2 a^6/a$, i.e., to $\rho^2 a^5$. In the case of a body of irregular shape, the electromagnetic criterion (13) or even the modified laws (18) or (19) which we have invoked, lead to a mass proportional to $\rho^2 l^5$, where l is a quantity comparable with the linear dimensions of the charge. Consequently, as our positive and negatively charged spheres approach each other and proceed to devour each other, the masses of the remnants diminish with great and increasing rapidity towards the end. The force between them, at any stage, is proportional to $(l^3\rho)(l^3\rho)/l^2$, i.e., to $\rho^2 l^4$, and consequently does not diminish with distance as rapidly as the mass. Thus, the rate of approach of the charges increases as they get smaller, although not as markedly as would be the case for two point masses of constant amount attracting according to the law of inverse squares.

The problem of charge annihilation which has been discussed is a relatively simple one, that of two spheres of equal size and density. The cases where the spheres differ in any respect result in rather curious situations which it is not worth while to discuss here in detail. The application of the force equation to the remnants of the charges during the annihilation assumes in general a form artificial in the highest degree, as in the case, for example, where a sphere sweeps through one of larger size and larger density and leaves a channel of diminished density in its wake. However, in all cases, the strict application of the principle leads to a radiation of Poynting energy through some completely surrounding surface σ, equal in total amount to the change of the volume integral of $(\mathbf{E}^2 + \mathbf{H}^2)/2$ resulting from the complete or partial annihilation of the charges.

Finally, it may be remarked that it is not the intention of this paper to maintain that the processes here sketched have reality in nature. The only purpose is to show in what sense mutual annihilation of charges with accompanying radiation of energy is to have meaning in classical electrodynamics if it is to have meaning in classical electrodynamics at all.

BARTOL RESEARCH FOUNDATION OF
THE FRANKLIN INSTITUTE,
SWARTHMORE, PA.,
July 15, 1931.

Electric Current from Wind Rotors. (*Compressed Air Magazine*, November 1931.) Windmills have been doing service for hundreds of years; and air turbines have been utilized for various purposes for decades. Now comes an effort to adapt the rotor principle in a manner that will make it possible to generate electric current on a worthwhile scale. An experimental unit is in course of construction at West Burlington, N. J.; and it seems that a number of public-service companies are financing the undertaking. How far those concerns will go in the erection of other plants will depend upon the results obtained at the one now in hand. The first unit, so it is reported, will involve an outlay of $100,000, but subsequently, if all goes well, the unit cost may be reduced to $40,000.

With the plant at West Burlington meeting the expectations of its inventor, it should generate about 1,000 kw. when operating in a 28-mile wind. We will not at this time make any effort to touch upon the details of the apparatus. The cost already stated plainly indicates that the plant is far from a mechanically simple one. In short, it bears no resemblance to the windmills and the air turbines that most of us have seen. In the last analysis, the effectiveness of the wind rotor as a source of electricity will, naturally, depend upon the availability of sufficient wind to operate it; and in the absence of trade winds, or of winds of dependable frequency, it is plain that a wind rotor electric plant can be used only as a standby for other plants that can be made to furnish energy at any time.

R.

Fastest Electric Car on Philadelphia Road. (*Popular Science Monthly*, December 1931.) What is declared to be the fastest electric car in the world has been designed for a thirteen-and-one-half mile suburban railroad between Philadelphia and Norristown, Pa. It is expected to develop a top speed of between eighty and one hundred miles an hour. Ten of the streamlined aluminum cars will soon be placed in passenger service. The fishlike shape, which reduces wind resistance at more than mile-a-minute speeds, was given hundreds of wind tunnel tests in the University of Michigan's aeronautical testing laboratory before the final design for actual service was chosen. Each car carries fifty-two passengers.

R.

THE STATUS OF CHROMIUM PLATING *

BY

WILLIAM BLUM, Ph.D.

I assume that in selecting the subjects of these lectures, your committee did not confine itself to the activities of Franklin! It would not be difficult however to justify the subject of almost any address to this Institute by an actual or potential interest of the "polygonal" Franklin (to paraphrase A. D. Little). In the present case it requires no great stretch of the imagination to visualize Franklin using chromium on his printing plates, especially those for Poor Richard's Almanac, the "best-seller" of that day. No doubt he would also have applied chromium to the plates for printing such paper money as the mother country permitted the colonies to issue. His new stove would have acquired added luster from chromium, and perhaps chromium plated lightning rods would have been fashionable!

Although the commercial applications of chromium plating have all been developed during the last decade, the process was discovered by Bunsen in 1854, and was accomplished from the present type of solution by Geuther in 1856. At his death in 1790, Franklin was therefore closer, chronologically, to the discovery of chromium plating than we are today.

The lapse of about seventy years between the discovery and the application of chromium plating is a good example of an important scientific fact or principle lying dormant until the time was ripe for it to be revived and used. The development of the automobile industry, especially to that stage where appearance became an important consideration, furnished a great opportunity for the application of chromium plating. This need was soon met by the numerous laboratories that have contributed to the refinement and adaptation of the early methods.

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* Presented at a meeting held Thursday November 5, 1931. Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

In the following effort to summarize the present status of chromium plating, no attempt will be made to discuss details of the process. Instead the aim will be to give a bird's-eye view of the methods and applications, and especially the limitations and probable future uses. Such a survey is especially appropriate at this time. The experience of the past five years has shown that chromium can be applied economically under commercial conditions, and that it has certain definite advantages. During the same period, however, there has been a marked development of chromium alloy steels, for example of the "stainless" type. At present these two methods of obtaining a lustrous, tarnish-resistant surface are in direct competition, especially in the automobile industry. Although it is too soon to predict the outcome, some of the factors that will undoubtedly influence the final choice will be discussed.

The following data and information have been derived from publications of many laboratories, including the Bureau of Standards, that have conducted studies on chromium plating. Without attempting to itemize such references, I wish to acknowledge my indebtedness to all those publications. The more frequent mention of the Bureau of Standards, and especially of its unpublished data, is due to my close association with this work. More detailed information will be found in the technical journals to which occasional references are made, and especially in the summaries published by R. Schneidewind.[1]

The rapid and phenomenal application of electrodeposited chromium must have depended upon some unique or unusual properties. In this connection it is necessary to distinguish between the published properties of chromium, most of which refer to cast metal of uncertain purity, and the properties of electrodeposited chromium, many of which have not been quantitatively measured. Some of these properties apparently depend upon the conditions of deposition or of subsequent treatment. Moreover, the chromium as deposited invariably contains hydrogen (and possibly oxygen), so that, strictly speaking, the measured properties are not those of

[1] University of Michigan *Engineering Research Bulletins*, No. 8 (1927) and No. 10 (1928), and "Circular Series," No. 3 (1930).

chromium itself. Nevertheless it is usually the properties of the chromium as deposited that determine its usefulness. These will be referred to in the following discussion unless otherwise specified.

The physical property of chromium that is most striking is its extreme *hardness*, which is the basis of the most varied though not the most extensive applications of chromium. It is difficult to define hardness, much less to measure and correlate it with possible applications in which resistance to wear is sought. Very recently R. Schneidewind [2] measured the hardness of electrodeposited chromium by different methods, and attempted to correlate the results. He concluded that certain approximate relations hold for the Brinell and Mohs hardness of annealed metals, including chromium. He did not, however, determine the relation of the hardness measurements to wear-resistance. Wear itself is such a complicated phenomenon that no simple general relation between wear and hardness is to be expected. Experience with gages and printing plates has shown that the scratch hardness is the most promising, but still only an approximate, criterion of wear-resistance, whenever the latter involves principally abrasion and not impact. H. K. Herschman [3] found however that, on gages, chromium deposits produced or treated under different conditions showed appreciable differences in wear when there were no corresponding differences in scratch hardness. Plated gages that were subsequently heated to 300° C. yielded the best service. Such treatment expels hydrogen and, according to Schneidewind, decreases the scratch hardness. It also reduces the brittleness, which is an important factor in wear. Chromium deposited at relatively high temperatures is less brittle and less porous, probably because of a lower hydrogen content. Recently A. Willink at Frankford Arsenal has deposited chromium at very high temperatures and current densities, upon gages and dies, with good results.

The *thermal expansivity* of chromium is relatively low, hence when chromium is applied to the common ferrous and

[2] "Hardness of Chromium as Determined by the Vickers-Brinell, Bierbaum and Mohs Methods." Trans. A. S. S. T., 19, p. 115, (Dec.) 1931.

[3] *B. S. Jour. Research*, 6, p. 295, 1931.

non-ferrous metals and subsequently heated, there is a marked tendency for detachment of the chromium. On cast iron this tendency is not so great however as to prevent the use of chromium plated glass-molds. For some preliminary determinations of the expansivity by P. Hidnert at the Bureau of Standards, a chromium tube was prepared by depositing the metal on a brass tube which was subsequently dissolved out with nitric acid. When the chromium tube was heated in air to about 500° C., it was slightly shorter than at room temperature. This surprising result does not signify a negative coefficient of expansion for chromium. It was obviously due to the contraction that accompanied the evolution of the large volume of hydrogen always contained in electrodeposited chromium. When the tube was cooled it was about one per cent. shorter than originally. On re-heating, it showed the normal expansion coefficient of about 7×10^{-6} from 20° to 100° C. and about 9×10^{-6} from 20° to 500° C. The density of the tube as deposited was about 6.93, which is considerably below the I. C. T. value [4] of 7.1.

These purely preliminary data indicate that there is a need for a comprehensive study of the factors which govern the hydrogen content of chromium, the form in which it is present, and its effects upon the physical and chemical properties of the deposits.

The *color* of chromium deposited from chromic acid is bluish, in marked contrast with the whiteness of silver and the yellow cast of nickel. Alone, the blue color is not displeasing, but when adjacent to silver it is sometimes objectionable. It is reported that chromium deposited from a chloride solution is whiter than the usual electrolytic chromium.

Owing to its resistance to tarnish, chromium possesses certain advantages over silver for exposed reflectors, although the *reflectivity* in the visible spectrum is only about 65 per cent., as compared with 90 per cent. for silver. In the ultraviolet range,[5] chromium reflects much better than silver and hence has been used on therapeutic apparatus.

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[4] International Critical Tables, Vol. 2, p. 456, 1927.

[5] W. W. Coblentz and R. Stair, *B. S. Jour. Research*, 2, p. 343, 1929.

While chromium has a high *electrical conductivity*, it is not especially suited for electrical contacts, where its hardness and tarnish resistance might appear advantageous. In practice it has been found that the resistance of chromium contacts is high and variable, and that gold contacts are much preferable. The high contact-resistance is no doubt due to the presence of a film of oxide, which also causes the passivity which will shortly be referred to.

Another property that is influenced by the oxide film is the *resistance to wetting*. Chromium surfaces are not readily wet by water or aqueous solutions, or even by molten metals. Hence difficulty is encountered in soldering to chromium.

The *coefficient of friction*, especially when a lubricant is present, was found to be relatively low, in a few unpublished experiments of W. H. Herschel of the Bureau of Standards. Chromium has not yet, however, found extensive use on bearing surfaces.

The most important chemical property of chromium is its *resistance to tarnish* when exposed to the atmosphere. Strictly speaking, this property is not due to resistance to oxidation, but to initial susceptibility to oxidation and to the formation of a continuous, transparent, insoluble film of oxide. The work of U. R. Evans and others indicates that this is the most plausible theory of the "passivity" of chromium and many other metals. This passivity of chromium is also manifested in its resistance to the action of oxidizing agents such as nitric acid, and to the deposition of copper upon the surface, to which later reference will be made. In the "active" state however, chromium is readily attacked by acids, as would be expected from its position close to zinc in the electromotive series.

Because of its resistance to tarnish, it has been often, but erroneously, assumed that chromium will prevent *corrosion* of underlying metals such as steel. This expectation could be realized only if the coating were impervious. Actually, however, electroplated coatings of moderate thickness of any metal are somewhat porous. In addition, chromium as usually deposited contains numerous fine cracks, caused by the contraction that accompanies the escape of hydrogen. If the chromium were in its active state, it would no doubt pro-

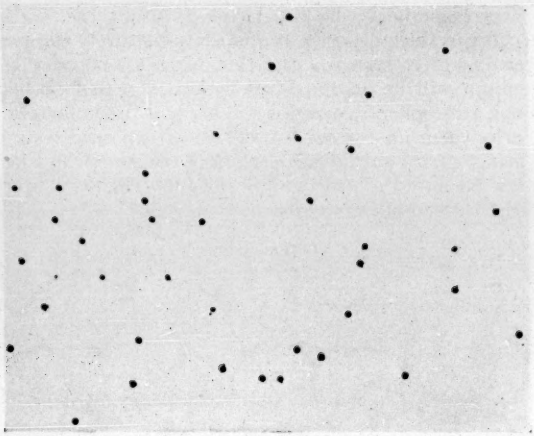
tect any steel that was exposed through pores, by "sacrificial" action, such as is usually exerted by zinc and cadmium on steel. But, as previously noted, chromium in contact with air is passive, that is, more noble than iron, and hence its presence may accelerate the corrosion of exposed iron instead of retarding it. It is unreasonable to expect chromium to stay bright in the air, and at the same time to corrode more readily than iron and thereby to protect the iron!

If then a chromium surface is desired on steel, together with protection against corrosion, it is necessary to apply either an impervious deposit of chromium, or one of some other metal prior to the chromium. The latter procedure has been adopted commercially. Present good practice, for example in the automobile industry, usually involves the deposition of nickel or of successive layers of copper and nickel, with a total thickness of about 0.001 inch (25 microns). The final nickel surface is buffed, and then a very thin coating of chromium, usually about 0.00002 inch (0.5 micron) is applied. In effect, the under-coatings protect the steel against corrosion, and the chromium protects the nickel against tarnish and maintains the desired high lustre.

Numerous authors have reported the existence of pores and cracks in chromium deposits, and Baker [6] and his associates have shown that under the usual conditions of deposition there is a minimum porosity when the chromium coating is about 0.00002 inch (0.5 micron) thick. In recent work [7] at the Bureau of Standards, the copper deposition test employed by Baker was made semi-quantitative. This test depends upon the fact that copper will not deposit as readily upon chromium as on other metals. Hence when a chromium plated metal is made the cathode in a copper bath, copper deposits only where there are pores or cracks, which are thereby delineated. By measuring either the average current, or the weight of copper deposited on a given area under specified conditions, the relative porosities can be roughly estimated. The results obtained by this method are illustrated in Figs. 1 to 4. They confirm the observations of Baker and also

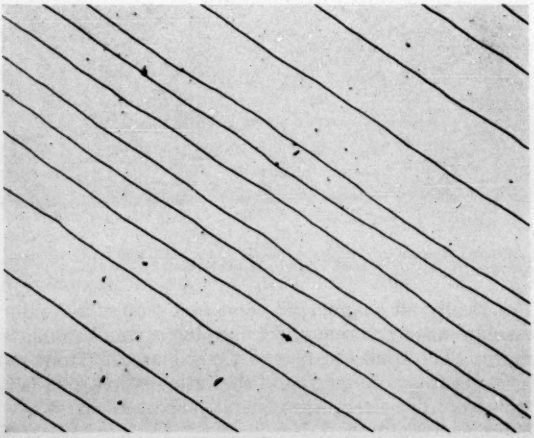
[6] E. M. Baker and W. L. Pinner, *Jour. Soc. Automotive Eng.*, **22**, p. 331, 1928.
E. M. Baker and A. M. Rente, *Trans. Am. Electrochem. Soc.*, **54**, p. 337, 1928.
[7] W. Blum, W. P. Barrows and A. Brenner, *B. S. Jour. Research*, p. 697, 1931.

FIG. 1.



Pores in thin chromium coatings on nickel (× 40). Thickness = 0.14 micron (0.000006 inch). Porosity = 25 ma./dm².

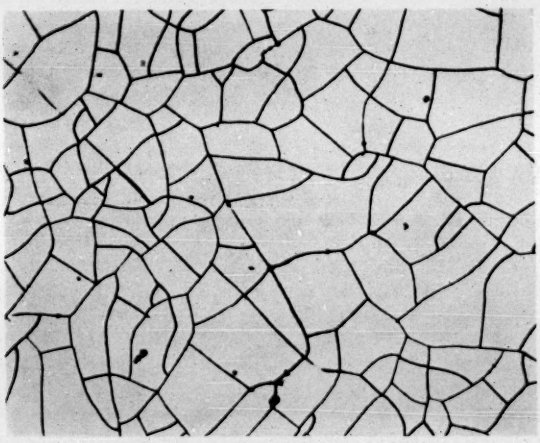
FIG. 2.



Parallel cracks in moderately thick chromium coatings on nickel (× 40). The cracks are perpendicular to the direction of last polishing of base metal. Thickness = 1.25 microns (0.00005 inch). Porosity = 180 ma./dm².

show that by depositing the chromium at higher temperatures and correspondingly high current densities, the cracking, especially of thick deposits, can be almost entirely eliminated. It remains to be seen whether this decreased porosity of the chromium will be accompanied by an increased protection against atmospheric corrosion. This and other factors will be included in an extensive program of exposure tests that are being carried out by coöperation of the American Electroplaters' Society, the American Society for Testing Materials, and the Bureau of Standards.

FIG. 3.

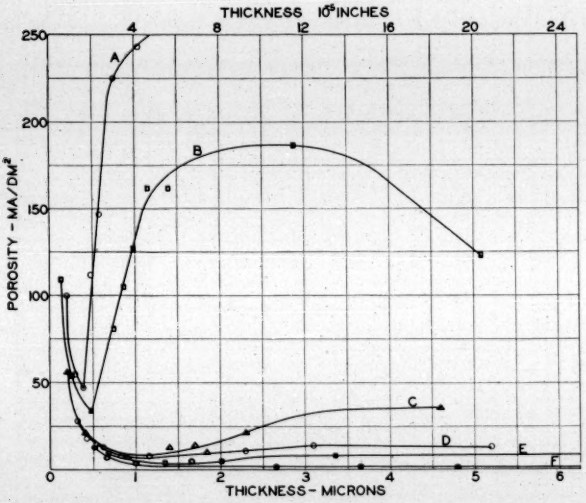


Random cracks in thick chromium coatings on nickel (× 100). Thickness = 2.75 microns (0.00011 inch). Porosity = 216 ma./dm².

Practically all commercial *chromium plating* is conducted from solutions of chromic acid containing a small amount of a sulphate. The concentration of CrO_3 is usually from 250 to 400 g/l (33 to 55 oz./gal.), and the ratio of CrO_3/SO_4 is from 100 to 200. The baths are generally operated at somewhat elevated temperatures, for example 45° C. (113° F.), and at relatively high current densities, such as 100 to 150 amperes

per sq. ft. (10 to 15 amp./dm².). Owing to the hardness of chromium, it is more difficult to buff than nickel and most other metals. Hence it is desirable to produce bright deposits of chromium that will not require buffing. Various researches have shown that for a given composition of solution,

FIG. 4.



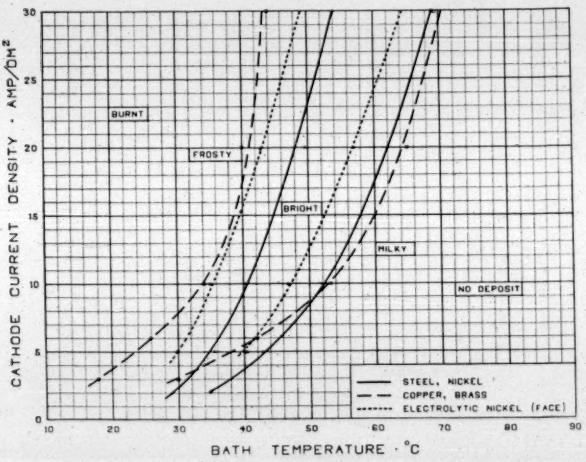
Porosity of chromium deposits on copper. Solution containing 250 g/l of CrO_3 and CrO_3/SO_4 = 100

	Temp. ° C.	Current Density amp./dm².
A	45	10
B	45	20
C	55	35
D	55	20
E	65	60
F	65	35

there is a limited range of temperature and current density within which bright deposits can be produced. As shown in Fig. 5, these ranges vary with the composition of the cathode metal (and also with its surface condition). Chromium plating therefore requires closer control than other plating, and in large plants the temperature is regulated automatically.

At any given temperature the character of deposit and also the cathode efficiency vary with the current density (Fig. 6). On irregularly shaped articles, the current density is never uniform. Consequently in plating such articles it is difficult to obtain a bright deposit over the entire surface, and the coating on the recessed part may be very thin or entirely lacking. This poor "*throwing power*" is a decided handicap and has taxed the patience of those engaged in commercial chromium

FIG. 5.

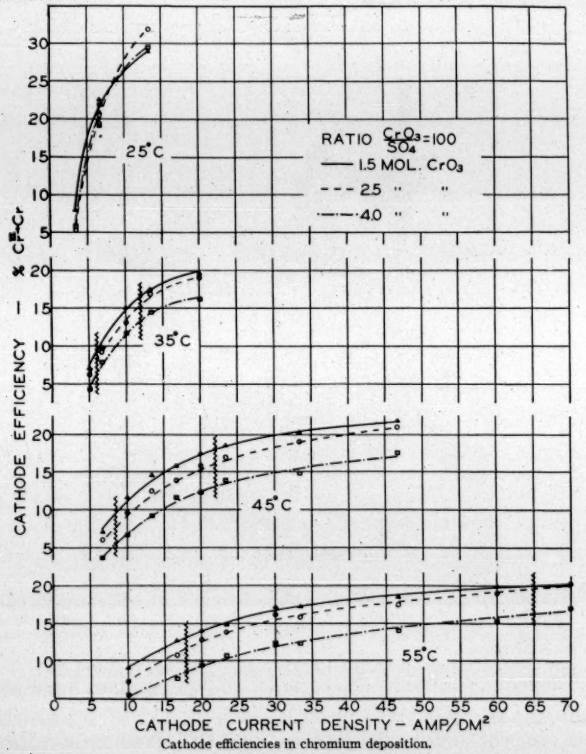


Plating ranges in chromium deposition. Solution containing 250 g/l CrO_3 and CrO_3/SO_4 = 100.

plating. Practically, it has been overcome more by ingenuity than by scientific research. The best results are obtained by expedients that make the cathode current density more uniform, for example by (1) increasing the distance between anodes and cathodes, (2) using parallel, concentric or auxiliary anodes, (3) attaching to the cathode a wire which acts as a "thief" and detracts the current from points tending to have excessive current densities, or (4) using non-conducting shields to reduce excessive current densities.

In general any condition that will make the cathode efficiencies more nearly equal with a change in current density, will improve the throwing power. Recent experiments [8] with

FIG. 6.



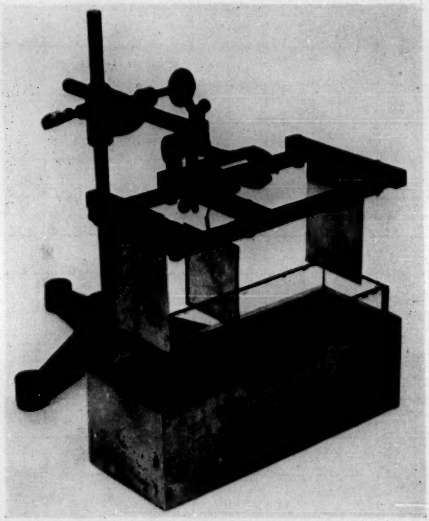
Cathode efficiencies in chromium deposition.

the throwing power box (Fig. 7) and also with bent cathodes (Figs. 8 and 9) have shown that slight improvements can be

[8] H. L. Farber and W. Blum, *B. S. Jour. Research*, **4**, p. 27, 1930.

made by using relatively high temperatures and current densities and a low sulphate content, that is, a high ratio of CrO_3/SO_4, such as 200 instead of 100. At best however such benefits are slight compared with those gained by favorable placing of the electrodes.

FIG. 7.

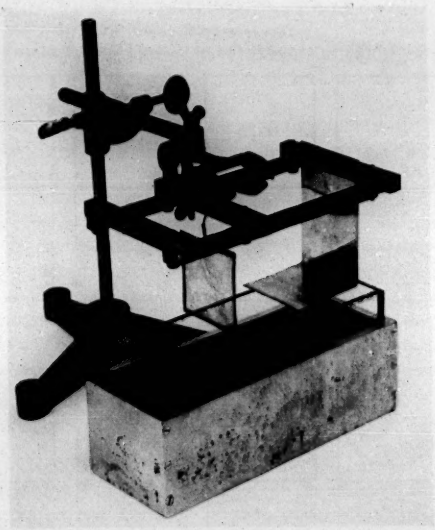


Throwing power box, as used for chromium plating. The throwing power is the percentage deviation of the ratio of the weights of metal on the two cathodes, to the primary distance ratio, which in this case is 2.

Unlike most other plating, this process employs insoluble *anodes*, usually of lead or iron. The former metal (especially six per cent. antimonial lead) is preferable, because at a lead surface any trivalent chromium formed at the cathode is more completely oxidized to hexavalent chromium than at an iron surface. Small concentrations of trivalent chromium are present in all used baths, and are not objectionable, but high concentrations increase the resistivity of the solution

and decrease the range for bright deposits. By the use of lead anodes as large as practicable the concentration of trivalent chromium can be kept within the desired limits. The metal deposited is replaced by the periodic addition of chromic acid to the bath.

FIG. 8.

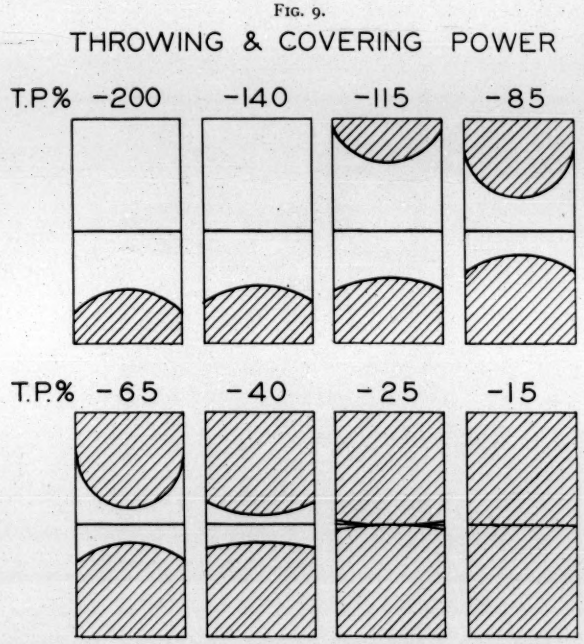


Apparatus for measuring the distribution of metal on bent cathodes.

The deposition of a given thickness of chromium *costs* more than of other common metals, in spite of the fact that, when purchased in the form of chromic acid, the chromium is cheaper than nickel. The cathode efficiency is usually less than fifteen per cent, and as the deposition involves reduction from a valence of six to zero, the quantity of current required for a given thickness is at least fifteen times that for nickel deposition, in which the valence is two and the cathode

efficiency is usually about ninety per cent. In some operations, especially at high temperatures and current densities, the bath voltage is as high as 10 volts, instead of the customary 6 volts used in nickel plating. As, however, the decorative chromium coatings are very thin, the actual cost of current is

FIG. 9.



Relation between numerical values of throwing power in chromium plating, and extent of covering of bent cathodes.

practically negligible, and certainly less than one-quarter of a cent per square foot. More skill and greater care must be used in chromium than in nickel plating. The adoption of chromium plating has not decreased the cost of the preliminary nickel plating, but has sometimes increased it, as it was found

necessary to improve the quality and the thickness of the nickel coating in order to realize the full benefits of chromium. In production, for example in the automobile industry, the cost of chromium plating must be added to that of the nickel plating formerly used. The increased expense is not prohibitive, and the total cost of a plated steel radiator shell is certainly much less than that of an alloy steel shell. The costs of polishing before and after the nickel plating are high as compared with the cost of the plating. However, the polishing of alloy steels is also expensive. The economy in plating results from a smaller total consumption of the relatively expensive nickel and chromium than are required in the alloy steels.

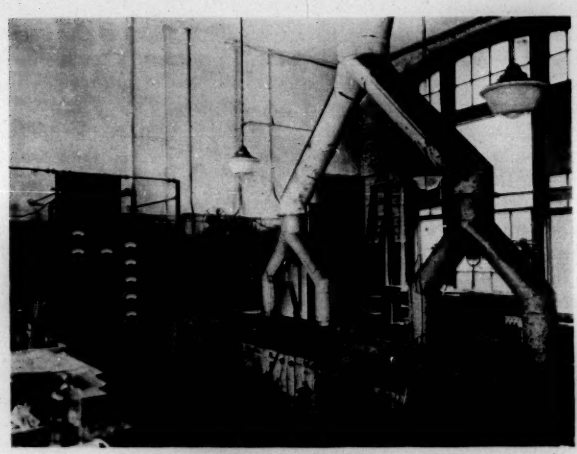
There is no adequate and satisfactory *theory* of the operation of the chromic acid bath, though numerous papers have been published on this subject. The one outstanding feature to be explained is the rôle of the sulphate radical (or other equivalent ion) in promoting the deposition of chromium. Such an effect may be due to a change in the static conditions, that is, the constitution of the bath; or in the dynamic conditions, especially the processes occurring in the cathode film. More quantitative data on both phases of the subject are needed in order to formulate a complete explanation. Such a theory of the chromic acid process might well lead to improvements in the deposition of chromium from its trivalent salts. Such deposition is now possible, but no process has thus far proved commercially successful. An obvious advantage of that method would be the lower current consumption required to deposit the chromium from the trivalent state, provided the cathode efficiency was at least equal to that now obtained. Such a process would probably involve the use of chromium anodes, which at present cost more than the chromium in chromic acid of comparable purity.

Soon after chromium plating became commercial, reports were received of *injurious effects* upon the workman. A joint investigation with the Public Health Service [9] showed that the injuries were mainly of two types, perforation of the nasal septum and sores on the hands or feet. It was demonstrated that with adequate ventilation the former difficulty could be

[9] J. J. Bloomfield and W. Blum, Public Health Reports, **43**, p. 2330, 1928.

completely eliminated. The most effective ventilation (Figs. 10 and 11) is accomplished by drawing a stream of air across the surface of the tank, into a slot in which the air velocity is at least 2000 linear feet per minute. The danger from "chrome holes" on hands or feet can be practically prevented by cleanliness, the use of rubber gloves, aprons and shoes, and frequent and thorough rinsing of floors. If chromic acid has entered a cut or sore, its effect can be counteracted by a solution containing a reducing agent, such as a sulphide or sulphite or a ferrous salt.

FIG. 10.



Ventilating flues for chromium baths.

The *applications* of chromium plating have become so numerous that their listing is impracticable. The two principal uses are for appearance and for wear resistance.

The *automobile industry* is the most extensive user of chromium plating. During the past five years almost every American make of car has used it for the metal trim, including radiator shells, lamp bodies, bumpers, hub caps and door

handles. In the haste to adopt chromium plating under the busy conditions existing in the automobile industry a few years ago, it is not surprising that unsatisfactory plating was occasionally produced, either through ignorance of the requirements of a good coating, or through inability or unwillingness to meet such requirements. In most cases the average quality and durability of the new plating was superior to the previous nickel finishes, but the public was led to expect

FIG. 11.

Horizontal slot through which fumes are drawn.

much from chromium and became more critical than formerly. Realization of the occasional defects led to two important steps in this industry, namely, studies to improve the quality of the nickel and chromium plating, and efforts to substitute alloy steels for plated steels. Recent reports indicate that some of the companies that have thus far depended entirely on plating, are likely to adopt alloy steels for articles subjected to severe abrasion, such as nuts and bolts, running boards and possibly bumpers. For the latter purpose, plain

steel "clad" with stainless steel has been suggested in order to reduce the cost. On the other hand it was recently reported that one large company is about to abandon its present use of stainless steel on radiators and lamps in favor of chromium plating. It is evident that the ultimate solution will be, not the exclusive adoption of one or the other product, but the application of each product to those parts for which it is best adapted. The ideal car should be like the "deacon's one hoss shay," in which all parts would deteriorate uniformly. Obviously there is no economic justification for using a relatively expensive product such as an alloy steel if its use produces a part that will greatly outlast the rest of the car, when the desired service can be obtained more cheaply by other methods.

Another important application of chromium plating is in the *plumbing* industry. In general the results have been favorable with, however, some unfortunate failures. The experience of the last five years indicates that it is generally preferable to plate brass fixtures with a relatively thick coating of nickel, followed by a thin film of chromium, rather than to apply a thick coating of chromium directly to the brass. It is at least probable however that the service from the less porous thick chromium deposits that can be produced at higher temperatures, might be more satisfactory than from those thus far employed. In recent years, especially in hotels and public buildings, many nickel-brass (German silver or nickel-silver) plumbing fixtures have been installed. If this alloy contains at least 18 per cent. of nickel, it is nearly white and is fairly resistant to tarnish. It can of course be polished indefinitely with no change in color, though the initial luster usually disappears. At present in most of the new large Government buildings, the fixtures are of 18 per cent. nickel-brass and are also chromium plated. It is too early to predict their life, but it is reasonable to assume that the chromium will insure a high luster for several years, after which the benefits of the nickel-brass will be retained, with no sharp color contrast where the base metal is exposed through wear of the chromium. One important precaution to be observed with all chromium plated fixtures is to protect them against the action of the hydrochloric acid that is often used to clean new tile

work. This acid attacks chromium rapidly and in some cases has ruined large installations. A heavy grease coating on the fixtures during this cleaning period will usually protect them.

The numerous other industries in which chromium has been applied for appearance generally involve plating on steel or brass under conditions similar to those mentioned, or on zinc or aluminum or their alloys. The latter metals are almost invariably plated with nickel before the chromium is applied. Deposits of chromium can be made directly on zinc or aluminum, but they are generally dull and have less protective value than the composite coatings.

Most of the applications of chromium to domestic equipment are ornamental and involve little or no contact with foods or beverages. Although no exhaustive studies have been made, the available information does not justify any grave apprehension regarding the use of chromium in contact with foods. Unpublished experiments of G. C. Spencer at the U. S. Bureau of Chemistry and Soils, indicate that most organic acids have no effect on chromium, though it is slightly attacked by citric and malic acids, which are present in many fruits. There is no evidence, however, that small amounts of chromium salts are toxic, and it is at least probable that they are as harmless as aluminum salts, which are present in most foods. The most serious objection to the use of chromium plating on actual food containers such as milk and ice-cream cans and cooking utensils, is that it is very difficult to produce coatings of chromium, or of chromium and nickel, that are and remain sufficiently impervious to prevent corrosion of the underlying steel or brass. Unless that is accomplished, the chromium plating is of little value, except to improve the initial appearance and thus reduce "sales resistance."

While the applications of chromium plating for appearance are more extensive than for *wear resistance*, the latter uses are more likely to increase in number and extent. As previously noted, the application of chromium for appearance almost invariably adds to the cost of manufacture. When, however chromium is applied to reduce wear and to prolong the useful life of a tool or device used in manufacturing, the cost of production is correspondingly decreased.

One of the most successful applications of the hardness of chromium is on measuring devices, such as dimensional gages.

At present many large plants are using chromium plated plug gages, which have a longer life than any other gages except those of tungsten carbide ("carballoy"), which is much more expensive, and more difficult to fabricate. One advantage of the plated gages is that when they become worn below the permissible tolerance, they may be salvaged by stripping off the chromium coating and replating them. In some plants just sufficient chromium is applied to afford the exact desired dimension, and in others an excess is applied and is removed by grinding and lapping. Plug gages can be uniformly plated by using simple precautions. Thread gages are likely to be changed in angle by plating unless special steps are taken, such as grinding the original gage to a predetermined angle other than the usual 60°.

In estimating the value of chromium plated gages to industry the considerable saving in the cost of replacing the gages may be a secondary factor. Far more important is the improvement in the average precision of the product. Before a gage is rejected, it must have approached or even slightly passed the specified tolerance, hence a certain proportion of the product may be correspondingly inaccurate. If now the average life of the gage is increased to three- or five-fold, the proportion of articles of questionable accuracy is reduced.

For several years the standard fifty-meter tapes used by the U. S. Coast and Geodetic Survey, have been plated with chromium for a short distance on each side of the gage marks. This coating has not only increased the permanence of these marks, but has also decreased the tendency for the formation of other scratches which might be confused with the gage marks.

Chromium has been successfully applied to forming-dies, including those used for drawing nickel and copper tubes, and those for shaping sheet metals, especially steel and brass. In addition to increasing the useful life of the dies, the chromium reduces the tendency of the metals to stick to the dies, and thus speeds up production.

Chromium is also very valuable on dies for molding plastic materials, such as rubber, bakelite, clay, and glass. Such applications depend on the resistance of the chromium not only

to abrasion, but also to chemical action, for example by the sulphur, phenol, or other constituents of the plastics.

Owing to the brittleness of chromium, it is not especially adapted for forming operations involving severe impact. If however the chromium coating is made relatively thin and very adherent, it may prove advantageous. Experiments at the U. S. Mint in Philadelphia showed that on the dies for stamping nickel coins a thick coating of chromium was detrimental, but a thin coating, for example about 0.0002 inch (5 microns), added materially to the life of the dies and to the quality of the coins.

Similarly on rolls for forming foil of soft metals, chromium has proved advantageous. It is doubtful however, whether it will withstand the heavy shock of rolling steel.

Chromium is valuable on tools like knives and saws that are used for cutting relatively soft but sometimes highly abrasive materials, such as wood, paper, fiber, asbestos, and rubber. It is of doubtful value on metal-cutting tools, as it is difficult to apply a thick enough deposit to be effective, without dulling the edge and also causing subsequent flaking of the chromium.

One of the outstanding applications of chromium is on printing plates, on which the wear is principally by abrasion. One of the first of such uses was at the U. S. Bureau of Engraving and Printing, where chromium has for years been applied to the plates employed for printing paper currency and postage stamps. In the present process the currency plates are electrolytically reproduced, and consist of electrodeposited iron with a surface layer of nickel. Upon the latter, a chromium coating with a thickness of about 0.0002 inch (5 microns) is applied. The useful life of the resultant plates is about four times that of the plain electrolytic plates, and twice that of the case-hardened steel plates that were formerly used exclusively for this printing. As the electrolytic plates, including the chromium, cost less than the steel plates, a marked economy has been effected. The impressions are also of superior quality. Chromium is applied directly to the steel plates used for printing stamps. These are stripped and replated several times, thus greatly increasing their useful life.

Chromium has also been used on electrotypes for printing long editions, for example of wrappers, labels, and cartons. The increase in service is of course variable, but in many cases it is at least five-fold. There is no justification for applying chromium to printing plates used for only small editions, or to those used under such press conditions that the plates fail through crushing or other deformation, against which the thin layer of chromium furnishes no protection.

While numerous uses of chromium have been suggested on moving parts of machinery, few such applications are extensive. On several makes of automobiles, chromium is applied to the piston pins, to protect them against both abrasion and corrosion. Chromium is also used on gears and shafts of small devices such as speedometers and other counting machines. One objection and obstacle to the more general use of chromium on bearing surfaces is its brittleness and consequent tendency to flake. It is at least probable that by depositing the chromium at higher temperatures or by heating the plated articles, this brittleness can be reduced.

One interesting application of chromium that involves moving parts, is on the interior of guns of both large and small caliber. At the Frankford Arsenal in Philadelphia methods adapted to hand rifles and machine guns have been developed, while at the Washington Navy Yard numerous six inch guns have been plated. In general the plating has resulted in an increased life and accuracy of the guns. Such coatings are usually not more than about 0.0004 inch (10 microns) thick.

Chromium plating has taken its place among the established plating processes and has ceased to be a novelty. It is probable that its use in the automobile industry for appearance has reached its peak, and that from now on the competition between chromium and the alloy steels will be one of give and take, depending upon such improvements in either process as affect the cost, convenience and durability. The same problems will be met in other industries where an attractive, permanent finish is required on metals.

Any large increase in the application of chromium plating will probably come in those industrial uses that have so far been barely touched. Improvements in the quality of the plating and in the design of equipment to utilize the advan-

tages of chromium will no doubt lead to many new applications for wear resistance. Another important and even more difficult but promising field is on equipment that is required to withstand both corrosion and abrasion, for example in the paper, textile and other chemical industries. The solution of many such problems will probably be found in the use of corrosion-resistant base metals, coated with chromium to protect them against abrasion.

Such problems as can not be solved by the use of chromium, may well yield to the application of other plated metals, including those that are not now commercially employed. The great progress in the plating industry during the past few decades, in both plants and research laboratories, augurs well for the future. Recent improvements in the deposition of the platinum metals and of tungsten show that electroplating will continue to meet new problems of industry, just as the advent of chromium plating has solved many such problems.

Some economists have recently blamed science for the present business crisis, and have urged a moratorium on scientific research. Even if so radical a suggestion were adopted, the full energy of scientists for many years could be employed to enable society to "catch up" with present scientific knowledge. Such efforts should be directed not only to teaching the industrial workers to apply science in their labors, but also to educating the general public to adapt its needs and wants to the new knowledge. In its lectures and publications, the Franklin Institute is contributing materially to such progress, and is following in the footsteps of Franklin, that first great American scientist, so aptly designated by Bernard Faÿ as "the apostle of modern times."

Height of the Heaviside Layer in Bengal. HRISHIKESH RAKSHIT. (*Phil. Mag.*, Nov. 1931.) It is well known that the upper region of the atmosphere, ionized and rendered conducting chiefly by the ultraviolet rays from the sun, is responsible for the bending of the radio waves round the surface of the earth. A knowledge of the nature and height of this region at different places on the surface of the earth, and a record of its variation with the season and with the hour of the day or night, are of considerable importance in the study of the propagation of the radio waves.

Experiments for the measurement of the height of the ionized layer of the upper atmosphere in Bengal are described. The average lowest height of the ionized layer, which is obtained near about sunset, for various evenings during the month of February 1931 has been found to be 60 km. The height gradually increases with the progress of the night, reaching on an average a value of 85 km. at about 11 p.m. The corresponding lowest height, as determined by Appleton and Ratcliffe, is about 90 km. The discrepancy between the two values is not at all surprising, considering the difference in the geographical positions and geophysical conditions of the two places of observation. The variation of the intensity of fading with the progress of the night can be divided into three types. These are described, and curves are drawn showing their general nature.

The observations made during the early hours of the morning generally indicated a gradual diminution in the height of the ionized layer as the hour of sunrise approached. It was also found that the intensity of fading approached zero at about 15 to 20 minutes after sunrise.

CHARACTERISTICS OF THE OSCILLOGRAPH-GALVANOMETER.

SOME PRACTICAL CONSIDERATIONS IN THE DESIGN AND APPLICATION OF THE OSCILLOGRAPH-GALVANOMETER VIBRATOR.

BY

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SYNOPSIS.

The paper describes the mechanical characteristics of the vibrator for any periodic phenomena, so that the mechanical error due to the vibrator may be eliminated from the oscillogram and the true phenomena determined. The paper establishes the best damping to use, when considered from the standpoint of vibrator response, lag, and transient characteristics. There is a method given to determine the vibrator resonance and damping, that may easily be applied in the field. Curves, from which all the above mentioned may be obtained, are supplied.

INTRODUCTION.

With the present materials available the oscillograph-vibrator, in its advance toward perfection, is approaching its ultimate limits in design as an instrument for instantaneous measurements. This small mechanical vibrating system has hundreds of applications, from the recording of the low frequency voltages produced by the human heart pulsation to that of the high frequencies of musical notes in talking movies, and from the recording of the slow moving train stresses in railroad bridge girders to the rapid firing of aeroplane engines. As the vibrator is made to respond to lower currents, the maximum frequency at which it will respond becomes smaller as well, and conversely, high frequency vibrators require larger currents to drive them. The limiting condition in design of a vibrator is to obtain the maximum sensitivity with the highest possible resonant frequency. There are three constants which determine vibrator sensitivity and resonance, namely: inertia, spring, and damping. Once the first two of these constants are set, they can be relied

upon throughout the life of the instrument, but the third, that of damping, on which the good performance of the instrument, throughout its range of frequency, finally depends, must, at various times, be determined and adjusted to suit the particular case in order to assure the most accurate results.

(1) As well as knowing the correct damping and its effect upon the test results at hand, it is often of great importance to know the vibrator resonant frequency and the effects resonance has upon the test results. The resonant frequency of the vibrator, when placed in damping fluid, is always lower and independent of the resonant frequency in air as usually quoted.

(2) The steepness of the wave front a vibrator is capable of recording, and the error in recording a steep wave front, are other important considerations.*

A FEW HIGH POINTS FROM THE THEORY OF VIBRATION.

In order that a few of the practical points in the theory of vibration may be brought out as they apply to the performance of an oscillograph vibrator, it is necessary to take a few equations as given by some of our noted authorities on the subject and use them here without a rederivation. The resultant vector equation for forced vibration applicable to a vibration system with one degree of freedom such as the bifilar oscillograph vibrator is

$$Iw^2\theta + nw\theta + k\theta = Bi/10,^{2,4,5,6} \qquad (1)$$

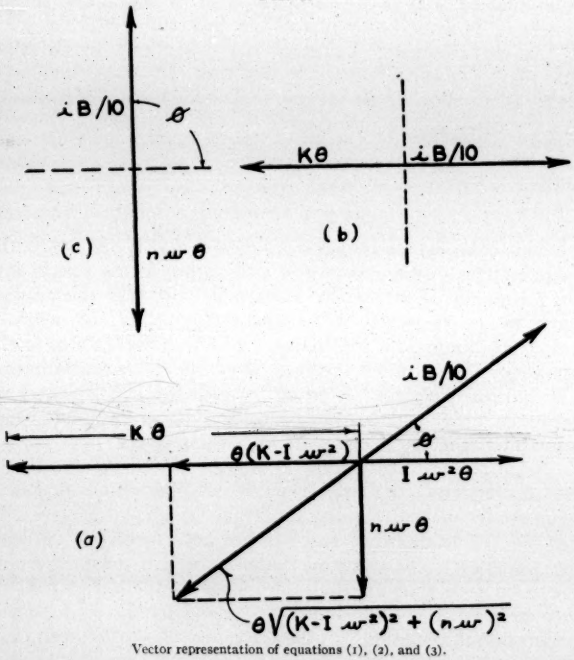
where I, n and k are inertia, damping and spring constants respectively and θ the deflection, w the angular frequency, B the flux density and i the current. The current determines the periodic force and may be a periodic function of a fundamental and all its harmonics; it may even be superimposed upon a direct current component. The inertia force ($Iw^2\theta$), varies as the frequency squared ($w = 2\pi f$), the damping force, (nw), varies as the first power of the frequency, and the spring or control force, ($k\theta$), is independent of the frequency.

* Consideration number (2) in the introduction the writer hopes to cover completely in a later paper.

The right hand side of the equation is the applied force per unit length of vibrator ribbon.

This equation may be represented vectorially as shown in Fig. 1, the inertia and spring forces exactly opposed, for, at

FIG. 1.



Vector representation of equations (1), (2), and (3).

the full deflected position of the vibrator, both are at a maximum but are 180 degrees displaced from each other, the damping force is a maximum 90 degrees later, therefore it is placed at right angles with the above two forces, and the applied force is exactly opposite to the resultant of the other three. When current is of zero frequency, equation 1 becomes

$$k\theta = Bi/10$$

and the vector diagram may be represented by Fig. 1-*b*. The *DC* sensitivity per unit length of vibrator ribbon is then

$$i/\theta = 10k/B \text{ (amperes per unit deflection).} \qquad (2)$$

The flux density (B) in any air gap has a limiting value, it is thus evident that if the ratio i/θ is to be small, the spring-constant (k) must be small and the deflection (θ) large.

As the frequency increases the other two forces come into play. Inasmuch as the spring-force ($k\theta$) does not change, there is a value of frequency that makes the inertia-force exactly equal to the spring-force

$$Iw^2\theta = k\theta$$

or

$$f_r = 1/2\pi \cdot \sqrt{k/I}. \qquad (3)$$

This is known as resonant frequency and is defined as the frequency at which the inertia and spring forces cancel and the frequency at which the vibrator lags behind the applied force by 90 degrees. The natural frequency, or what is sometimes known as the frequency of free oscillation, is the frequency of maximum response, and changes with damping. The natural frequency is equal to resonant frequency at no damping, only, and becomes zero at critical damping while resonant frequency is constant for all dampings.

From the vector diagram, it is seen that these two forces are 180 degrees out of phase with each other and cancel at resonant frequency, leaving the vector diagram as shown in Fig. 1-*c*. The damping force is the only force left to limit the deflection of the vibrator, therefore, with a constant input, it can be seen that at small dampings the deflection will be extremely large.

Observing equation (3), it can be seen that the inertia (I) which includes the mass, must be extremely small to make (f_r) large, for, from equation (2) it is shown that (k) must be small to make the current input (i) small. Therefore it can be seen why vibrators are necessarily small and delicate.

PHASE-LAG OF THE VIBRATOR.

Angle ϕ shown in Fig. 1 is the angle the applied force is leading that of inertia or deflection. In that the deflection

is always in phase with the vibrator, ϕ is the angle the vibrator lags behind the applied force. At resonant frequency ϕ is 90 degrees as given in Fig. 1-*c*. The variation of ϕ between zero and resonant frequency is given by equation

$$\tan \phi = nw/(k - nw^2). \qquad (4a)$$

This equation is hard to plot in this form but Mr. Irwin puts the above equation in another form by substituting p, for the ratio (w/w_r) of the actual frequency to that of resonance, and N, for the ratio (n/n_c) of existing damping to that of critical. Critical damping is the damping at which the vibrating system ceases to have free oscillation, that is, the damping has just reached the value where the deflections stop the instant the applied force is removed. Critical damping is given by equation

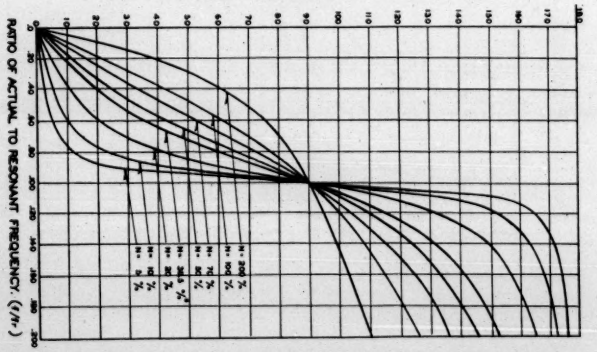
$$n_c = 2\sqrt{Ik}. \qquad (5)$$

Then equation (4*a*) may be written

$$\tan \phi = 2Np/(1 - p^2). \qquad (4)$$

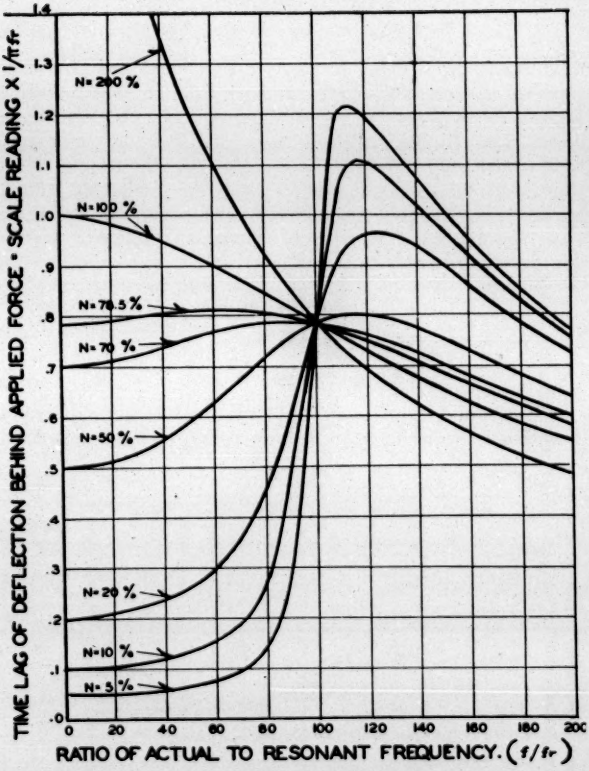
Curves in Fig. 2 show the angle lag for any vibrator as it varies with frequency and damping. These curves are

FIG. 2.



Curves showing angle lag of the deflection behind the applied force as it varies with frequency and damping.

FIG. 3.



Curves showing the actual time lag, in seconds, of the oscillograph behind the applied force, as it varies with frequency.

plotted from equation 4, by assigning various values to N and by varying the frequency f.

In consideration of the best damping for a vibrator, these curves give much information. If, for instance, with 10 per cent. damping and a fundamental frequency of 30 per cent. of resonance, from the curve the lag is 4 degrees, the third harmonic is 90 per cent. of resonance and the lag is 45 degrees. The actual time lag is almost 4 times greater for the third harmonic than for the fundamental. From the above case, there is no question but that a badly distorted wave would be reproduced by a vibrator. From the standpoint of lag, damping near 70 per cent. will reproduce the most accurate wave form.

The phase lag of a vibrator in degrees may be converted into time lag in seconds, and the curves replotted as shown in Fig. 3. The new curves show very plainly that a damping between 75 and 80 per cent. gives the nearest to a constant time lag for a wave regardless of the number harmonics, provided the frequency of the harmonics does not run much over resonance. These curves also point out that the time lag at resonance is independent of damping but as the frequency approaches zero the time lag becomes directly proportional to the damping.

VIBRATOR RESPONSE.

The vibrator response characteristic is the change in current sensitivity with frequency. This characteristic curve is taken by holding the applied force or current constant, varying the frequency and measuring the deflection of the vibrator. The deflection at zero frequency is taken as unity and all other deflections are taken as a ratio to that at DC.

The theoretical vibrator response curves may be calculated as follows: With reference to the vector diagram in Fig. 1-a, the vector of the applied force is equal to the resultant of the other three vectors

$$Bi/10 = \sqrt{(nw\theta)^2 + (k\theta - Mw^2\theta)^2};$$

when, as stated above, the left member of the equation is unity, then

$$1 = \theta\sqrt{n^2w^2 + k^2 - 2Mkw^2 + M^2w^4}.$$

By substituting the values of critical damping (n_c) and the angular frequency (w_r) in terms of I and K as given by equations (3) and (5) and solving for the deflection

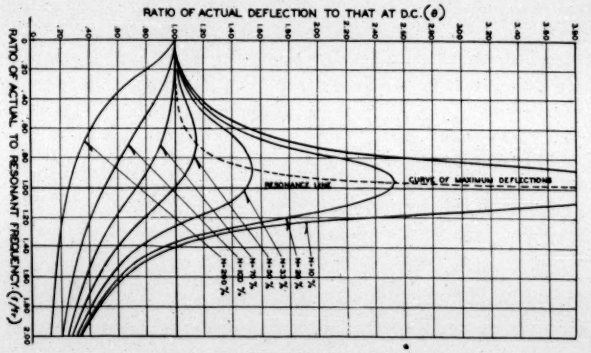
$$\theta = 1/\sqrt{p^4 + p^2(4N^2 - 2) + 1}^{5} \qquad (6)$$

where p is the frequency ratio (w/w_r) and N is the damping ratio (n/n_c). At resonance this equation (6) becomes

$$\theta = 1/2N. \qquad (7)$$

By assigning values to N, varying the frequency ratio, solving for the deflection, the response curves may be drawn from equation (6) as shown in Fig. 4.

FIG. 4.


RATIO OF ACTUAL DEFLECTION TO THAT AT D.C. (θ)
RATIO OF ACTUAL TO RESONANT FREQUENCY (p/w)
RESONANCE LINE
CURVE OF MAXIMUM DEFLECTION

Curves showing response characteristics of an oscillograph vibrator, as it varies with frequency and damping.

From the curves, the damping that gives the most constant response throughout the widest range of frequency is near 70 per cent. damping. Sixty per cent. damping is considered a very good response characteristic. At 70.7 per cent. damping and greater, the response never goes beyond unit deflection, for any frequency.

THE EFFECT OF DAMPING FLUID ON VIBRATOR RESONANT FREQUENCY.

The principle disadvantage to a mechanical vibrating system employing oil damping is that, as soon as the me-

chanical resistance is added to the system to dampen it, the resonant frequency of the system is reduced. Irwin said, "Actually, if the undamped instrument be immersed in oil to secure damping, its resonant frequency is reduced owing to the inertia of the oil put in motion." [5] There has been no mathematical formula developed to calculate the amount of oil put in motion. For, when an object is oscillating in oil at such a terrific speed as an oscillograph vibrator, it is difficult to predict the exact manner in which the oil adheres to the system to add its inertia. But there are some things that test seems to indicate, that may just as well be stated here.

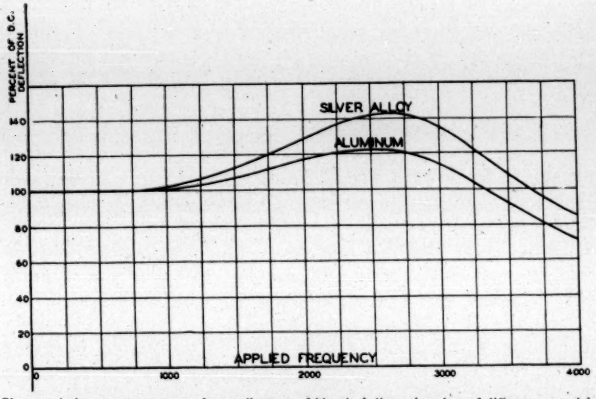
Irwin also stated that the reduction in resonant frequency due to the added inertia of oil, was about one-third or the resonance in oil was two-thirds of that in air. Tests made by the writer and some of his associates show that the reduction in resonant frequency of a vibrator when immersed in oil, utilizing silver ribbons, was more than one-third. The reduction was greater when the specific weight of the vibrator ribbon was less. For instance, the silver vibrator ribbon commonly used in oscillographs gives a resonant frequency in oil near 57 per cent. of that in air, for molybdenum ribbon 55 per cent. of that in air, and for aluminum vibrator ribbon the oil resonant frequency is 42 per cent. of that in air.

Another interesting thing is, no matter what size, shape and spacing these vibrator wires are given, as far as the accuracy of the tests would indicate, the relation between oil and air resonance remained the same for the given wire material. Furthermore and within the accuracy of the tests, the kind of oil or its viscosity did not change the ratio between oil and air resonance for the given ribbon material.

A common failing is to state the resonant frequency of a vibrator as found in air, which is often misleading as to what to expect of the vibrator when immersed in oil. A good example is as follows: Of two vibrators of practically the same sensitivity, one of silver with a resonance in air of 5,600 cy. per sec. and another of aluminum with an air resonance of 8,100 cy. per sec., proved, by test under exactly the same conditions, to have practically the same resonant frequency in oil. The silver vibrator resonance in oil being 3,200 cy.

per sec. and the aluminum 3,400 cy. per sec. The response curves of these two vibrators are given on Fig. 5. The aluminum vibrator is damped more in the same oil than is the silver vibrator. Actually there is but little gained by using the so called high frequency aluminum vibrator when it must be damped in oil.

FIG. 5.



Characteristic response curves, of two vibrators of identical dimensions but of different materials and taken under similar circumstances. The resonant frequency in air was, aluminum vibrator 8,100 cycles per second, and the silver alloy 5,600 cycles per second.

DAMPING FROM A SINGLE OSCILLOGRAM.

The damping of the oscillograph vibrators may be determined by any of several different methods. The method requiring the least equipment and the one that might easily be used in the field, employs the theory of the logarithmic decrement, or the rate of dying down in amplitude of the free oscillation.

Any load applied suddenly to a vibrating system will cause the system to oscillate about a new position of equilibrium at a frequency depending upon the amount of damping and the resonant frequency of the system. A common illustration of the above condition is manifested in an indicating instrument when an electric switch is closed allowing a rush of current to flow through the instrument. The needle of the instrument, in attempting to indicate the current flow,

will be seen oscillating about the position of steady deflection until it finally comes to rest. The same condition occurs in an oscillograph vibrator when direct current is suddenly put through it, the beam from the vibrator will be seen oscillating about its new position. It is considered better practice to break circuit and then let the instrument oscillate about its own position of equilibrium; then the error due to the inductance of the circuit is eliminated.

The rate of dying down in amplitude of these free oscillations is a geometric progression and the rate of decrease varies inversely as the damping. The amplitude of the first over-swing will be the same percentage of the steady deflection as the first under-swing (which is the first swing back toward the zero position) is of the first over-swing and so on.[7] Then by knowing the percentage, the first over-swing is of the steady deflection, all other swings can be predetermined, and the exponential curve, showing the rate of decrease in amplitudes, can be drawn. The curve is of the form

$$x = X_o e^{-a}\ ^{7}$$

where x is the amplitude of the particular oscillation, X_o is the steady deflection, and a is the decrement of oscillation.

From the curve and equation, the damping of the vibrating system can be determined, which is too laborious to work out every time. But if we go through the calculation once and for all and plot a curve between the first over-swing and damping, one need only to find what percentage the first over-swing is of the steady deflection, then from the curve, the damping may be found. Before the curve can be drawn, a few relations must first be established.

The above equation can be rearranged and put in a form to express the amplitude of the first half cycle of free oscillation in terms of the deflection ratio at resonant and zero frequency, with a constant energy input into the vibrating system. The equation will appear as

$$x = X_o e^{-\pi/\sqrt{4\theta^2-1}}. \qquad (8)$$

Where θ is the ratio of the deflections at zero and resonant frequencies (θ_r/θ_o), in terms of the damping ratio. It is as follows:

$$\theta = \theta_r/\theta_o = 1/2N = n_c/2n. \qquad (7)$$

To solve for the curve, assign values to N, ranging from .05 to 1.00 and solve for θ and X, which are given in Table I and plotted in Fig. 6. This curve can be used for any

FIG. 6.

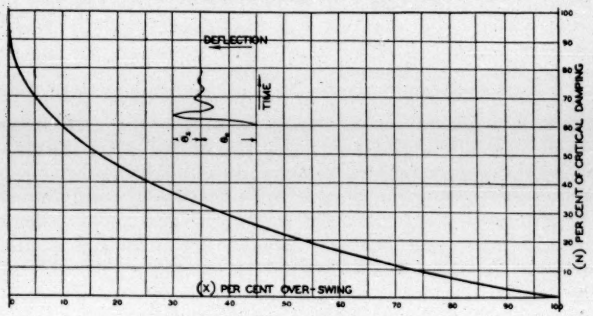


Fig. 6 - Damping of an instrument from the transient or overswing, (per cent overswing X = (θ_s/θ_0) 100

Damping of an instrument from the transient or over-swing (per cent. over-swing, $x = (\theta_s/\theta_0)100$.

vibrating system of one degree of freedom providing the damping is proportional to the velocity of vibration.

Incidentally, the curve is also adaptable to indicating instruments. The task is very simple; adjust the current flow until the maximum needle swing just reaches full scale, then when the needle comes to rest the percentage of the over-swing can easily be determined and damping is then merely taken from the curve.

TABLE I.

θ	N	X	θ	N	X
10.00	0.05	.854	1.25	0.40	.253
5.00	0.10	.728	1.00	0.50	.163
2.50	0.20	.526	0.83	0.60	.095
3.33	0.15	.625	0.71	0.70	.046
2.00	0.25	.444	0.59	0.85	.006
1.50	0.33	.329	0.50	1.00	.000

RESONANT FREQUENCY FROM A SINGLE OSCILLOGRAM.

The vibrator resonant frequency in oil is not an easy thing to find experimentally for it does not occur at the maximum response. At low dampings, the point of resonance occurs at a frequency near enough to the maximum response to be considered as the resonant frequency. For instance at 20 per cent. damping, the maximum response occurs at a frequency of 96 per cent. of resonant frequency. But at dampings commonly used in oscillograph galvanometers the maximum response is too far displaced from resonance to be considered as such. For instance, at 50 per cent. damping, the maximum response occurs at 71 per cent. of resonant frequency. An approximate method to find resonance, which is usually as accurate as the test itself, is then by direct comparison of the actual response curve with the theoretical curves including the curve of maximum deflection, which are shown in Fig. 4.

Another method to find the resonant frequency of a vibrator in oil, and one that needs no variable frequency source for a response curve, is to calculate it from the frequency of free oscillation of the vibrator at the given damping. When a vibrator is suddenly deflected, as described in section 5, it will oscillate about a new position at frequency as given by the following equation

$$f = (1/2\pi \cdot \sqrt{k/I})\ \sqrt{1 - N^2},^2 \qquad (10)$$

which is the frequency of free oscillation and $N^2 = n^2/4IK$. This equation approaches equation (3) in the first section, as the damping (N) approaches zero. By use of equation (3), equation (8) may be written

$$f = f_r\sqrt{1 - N^2}. \qquad (9)$$

This equation may be used to find the vibrator resonant frequency when damped. By suddenly applying a direct current to the vibrator, it will oscillate about a new position, as shown in Fig. 6. The per cent. overswing of the first half cycle will give value of N from Fig. 6. The length of a cycle of free oscillation into the film speed will give the frequency of free oscillation f, in equation 9. Then, the only unknown left in equation (9) is the resonant frequency f_r, which can then be solved.

CONCLUSIONS.

The best damping to give a vibrator, as summarized by the vibrator lag, response, and transient characteristics, is 70 per cent. of critical.

The complete characteristics of a vibrator can be obtained from a single oscillogram as follows:

First, find the damping and resonant frequency from the oscillogram curve.

Second, calculate the lag and response curves by use of equations (4) and (6).

Then any periodic phenomena taken with this vibrator may be analyzed and corrected for the recording error due to the vibrator.

APPENDIX.

The deflection of free oscillation at any time after the transient is given by

$$x = X_o e^a,^7 \tag{1}$$

where X_o is the steady deflection and a is the decrement of free oscillation,

$$a = -nt/2I. \tag{2}$$

n is the existing damping, t is the time for one complete oscillation, and I is the (moment of) inertia of the oscillating system.

To find the value of the deflection over-swing at end of the first half cycle, substitute in the time for one-half cycle of free oscillation,

$$t/2 = 1/2f = \pi/\sqrt{k/I - n^2/4I^2},^{7,2} \tag{3}$$

where k is the spring constant. N, the damping ratio

$$N = n/n_c = n/2\sqrt{kI} \tag{4}$$

where n_c is critical damping.

By use of equations (2), (3), and (4), equation (1) becomes

$$x = X_o e^{-\pi N/\sqrt{1-N^2}}. \tag{5}$$

The deflection at resonance is

$$\theta_r = 1/2N.$$

(Equation (7) of the paper.)

Then equation (5) may be written

$$x = X_o e^{-\pi/\sqrt{4\theta^2-1}}. \qquad (6)$$

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Some Physical Characteristics of Speech and Music. HARVEY FLETCHER. (*Jl. Acoust. Soc. Am.*, Oct. 1931.) Kinematic and statistical descriptions of the physical aspects of speech and music are given in this paper. As the speech or music proceeds, the kinematic description consists in giving the principal melodic stream, namely, the pitch variation and also the intensity and the quality variations. For speech and song, the quality changes are principally described by giving, besides the main melodic stream, two secondary melodic streams corresponding, respectively, to the resonant pitches of the throat and mouth cavities. To this must also be added the positions of the stops and the high pitched components of the fricative consonant sounds as functions of the time. The statistical description consists in giving the average, the peak, and the probable variations of the power involved as the various kinds of speech and music proceed. These general ideas are illustrated by numerous experimental data taken by various instrumental devices which have been evolved in the Laboratories during the past fifteen years.

GENERAL THEORY OF THE EFFECT OF INHOMOGENEITY ON THE BREAKDOWN VOLTAGE OF SYSTEMS OF INSULATION.

BY

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In previous papers the theory of the effect of insulation variability on the breakdown voltage of systems of parallel-connected units of insulation was developed [1] and comparisons made with the results of experiment.[2] The following is an extension of the theory to more complicated systems of series and parallel-connected units.

The results of the above mentioned papers are embodied in the following equation,

$$\bar{E}_N = \bar{E}_1\left[1 - S_1 \cdot \text{erf}^{-1}\left(\frac{N-1}{2N}\right)\right], \qquad (1)$$

which gives the expected breakdown voltage $\bar{E}_N$ of a system of N parallel-connected units of insulation in terms of the mean breakdown voltage of the units, $\bar{E}_1$, the number of parallel-connected units, N, and the coefficient of variability of the units, S_1, which is obtained from tests of samples. The symbol erf^{-1} represents the inverse error function. Its value may be obtained from tables just as are the values of sine and sine^{-1}. The equation shows that the breakdown voltage of such systems decreases linearly with increase in variability of the units composing it. The breakdown voltage of a system also decreases as its size increases, that is, as more units of insulation are connected in parallel.

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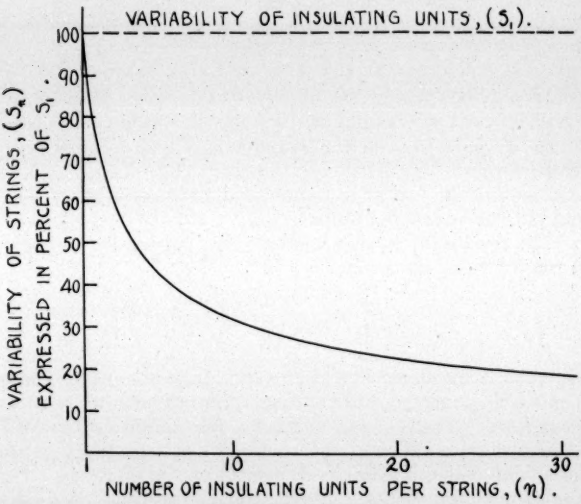
[1] "Breakdown Voltage as a Function of Electrode Area and Dielectric Homogeneity," JOUR. FRANK. INST., June 1931.

[2] "Insulation Variability, Its Influence in Determining Breakdown Voltages." Presented at Summer Convention, A. I. E. E. in June 1931.

The more complicated case of series-parallel connections of units may be approached in the following way. Consider a large "lot" of insulating "units." This lot is characterized by the mean breakdown voltage of the units $\bar{E}_1$, and their coefficient of variability S_1. The coefficient S_1, measures the "inhomogeneity" of the lot. A decrease in S_1 corresponds to

FIG. 1.



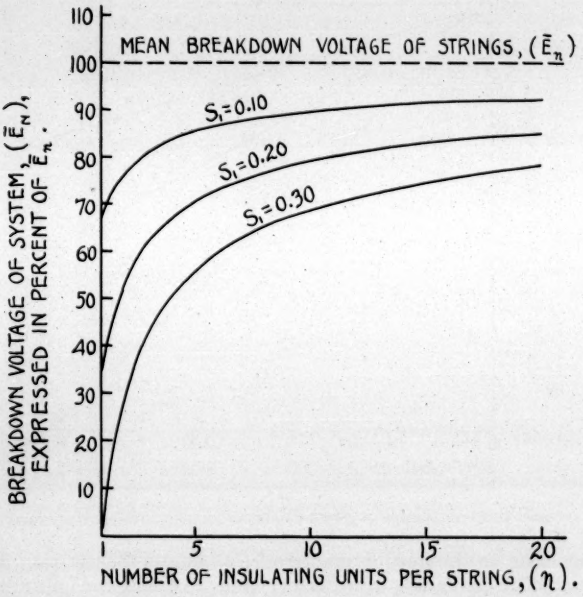
Curve showing the decrease in variability of strings brought about by increasing the number of insulating units per string.

an increase in homogeneity. A value of S_1 equal to zero would mean perfect homogeneity. Next, suppose that from this lot groups consisting of n units each are selected at random and connected in series in "strings" of n units per string. The mean breakdown voltage of such strings, denoted by the symbol $\bar{E}_n$, will equal $n\bar{E}$, if uniform voltage distribution along the string is assumed. The variability of such strings, however, denoted by the symbol S_n, will not be the same as that

of the units composing them, but will be less. It follows from modern sampling theory that the variability of the strings should be proportional to the variability of the unit insulators and inversely proportional to the square root of the number of unit insulators per string, in accordance with the following equation.

$$S_n = \frac{S_1}{\sqrt{n}}. \qquad (2)$$

FIG. 2.

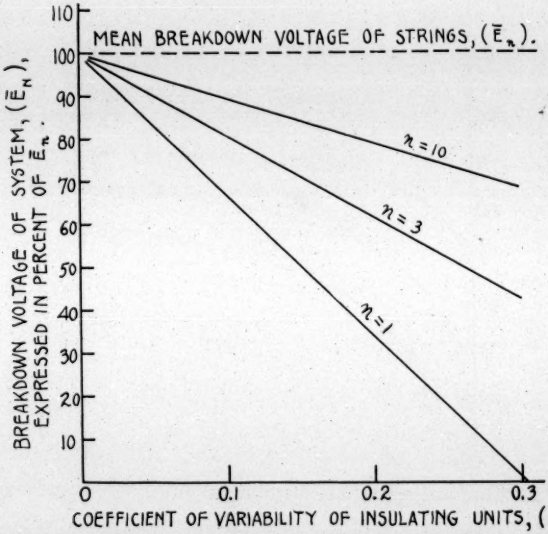


Curves showing the effect of increasing the number of units per string, for different grades of insulating units, for systems containing 1000 strings.

The relation is shown graphically in Fig. 1. This relation has important consequences. It immediately suggests a means for reducing variability and thus increasing the strength

of insulating systems. Next, assume these strings connected in parallel, as in overhead transmission lines, for example, forming a "system" of N strings in parallel. The "mean," "expected," or "long run" breakdown voltage of such systems, denoted by the symbol $\bar{E}_N$, will not be as great as that of the strings composing it, but will be less, the amount de-

FIG. 3.



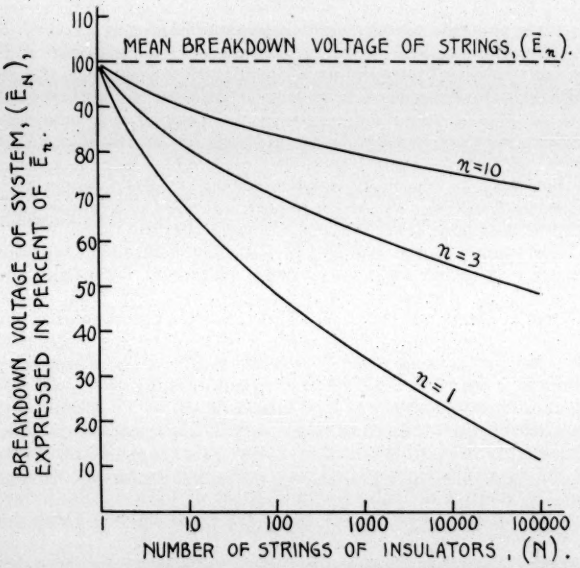
Curves showing decrease in system breakdown voltage with increase in variability of insulating units, for different numbers of units per string, for systems containing 1000 strings.

pending on the four parameters N, n, $\bar{E}_1$ and S_1. It may be found by combining equations (1) and (2) giving the following more general equation,

$$\bar{E}_N = \bar{E}_n\left[1 - \left(\frac{S_1}{\sqrt{n}}\right)\operatorname{erf}^{-1}\left(\frac{N-1}{2N}\right)\right]$$
$$= n\bar{E}_1\left[1 - \left(\frac{S_1}{\sqrt{n}}\right)\operatorname{erf}^{-1}\left(\frac{N-1}{2N}\right)\right]. \qquad (3)$$

The way in which $\bar{E}_N$ depends upon the four parameters N, n, $\bar{E}_1$ and S_1 is shown graphically in the accompanying sets of curves. These curves were obtained by fixing one parameter at a time and allowing the others to take on different values.

FIG. 4.



Curves showing breakdown voltage of system as a function of the number of strings connected in parallel, for different numbers of insulating units per string for units with variability. $S_1 = 0.20$.

$\bar{E}_N$ has been expressed, in each case, in per cent. of $\bar{E}_n$, thus showing how far the system breakdown voltage departs from, or approaches, that of the strings comprising it—a value which naturally constitutes the upper limit.

Light Signals in Aviation and Navigation. I. LANGMUIR AND W. F. WESTENDORP. (*Physics*, Nov. 1931.) Laboratory experiments have been devised to make measurements of the visibility of light signals under conditions essentially similar to those encountered by the aviator or the navigator. Data have been collected on the direct visibility of flashing point sources of light of different colors, flash lengths and intervals, against different backgrounds; the time it takes to locate a visible beacon was studied as a function of the beacon intensity and frequency of flashing. The threshold candle power C required for visibility of a point source at distance D (cm.) against a background of brightness H (candles $\times$ cm.$^{-2}$) is given by the empirical equation $C/D^2 = 3.5 \times 10^{-9}\ H^{\frac{1}{2}}$. Colored point sources were not found to be useful except in the case of red lights with background intensities above moonlight. For an airplane approaching a beacon it is advantageous to use frequencies of flashing as high as 12 to 30 per minute, although with exceptionally clear atmosphere, lower frequencies may be better.

In a study of the visibility of flashes of diffuse light superimposed on a steady white background, white light flashes gave the best results. The sensitivity of the eye to light from point sources is from 10,000 to 170,000 times as great as from diffuse sources, this range corresponding to an increase in background brightness from 0.1 starlight up to moonlight. A selective differential photoelectric receiver is described which detects signals of *modulated diffuse light* of an intensity of only 4×10^{-11} candles $\times$ cm.$^{-2}$. This sensitivity is independent of the steady background brightness up to 100 times moonlight, and is from 6 to 13,000 times as great as that of the eye in the range of background intensity from darkness up to moonlight.

The greatest difficulty in transmission of light signals through fog lies in the loss of advantages of the point source. Dense fog may increase the distances at which *diffuse light signals* may be detected. The range depends to a considerable extent on the reflectivity of the ground. A theoretical treatment of the diffusion of light through fog, based on the scattering of the light rays by fog particles, indicates that airplanes can be guided through fog at distances of several miles by means of diffuse modulated light acting on a differential photoelectric receiver.

MASS AND ENERGY.

BY

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In what follows, no attempt will be made to discuss the relation between mass and energy from the standpoint of the wave mechanics. That problem has been attacked by the author [1] and will be published elsewhere. The present discussion concerns the problem as viewed from classical standpoints. The relation,

$$\text{mass} = \text{energy}/c^2, \tag{1}$$

has assumed a very profound significance in modern physics, but it suffers from the advantage of being so well known that a considerable bracing of one's courage is necessary to give one confidence to ask where it comes from. One feels that the thunderbolts of Jove will descent upon his head for questioning, in the smallest degree, its complete respectability. Any suggestion of an inquiry as to its origin is apt to be met with the answer that it "comes from the theory of relativity." And if one is not reduced to submission by that statement and proceeds to inquire how it comes from the theory of relativity, he is apt to encounter some such naive argument as the following:

Let us assume that the equations of motion of a particle represent a relation between the acceleration 4-vector

$$k\frac{d}{dt}k\mathbf{w}, \qquad k\frac{d}{dt}(kic),$$

where

$$k \equiv (1 - w^2/c^2)^{-1/2}$$

[1] In an address, "The Relation Between Mass and Energy on Wave Mechanics," given before The American Physical Society at the Pasadena meeting, June 16, 1931.

and some other 4-vector,

$$\mathbf{K}, \qquad i(\mathbf{K}\cdot\mathbf{w})/c$$

which, as its form indicates, is perpendicular to the velocity 4-vector in the four-dimensional sense. The relations,

$$\mathbf{K} = k\frac{d}{dt}\frac{m_0\mathbf{w}}{\sqrt{1 - w^2/c^2}}, \tag{2}$$

$$(\mathbf{K}\cdot\mathbf{w}) = k\frac{d}{dt}\frac{m_0c^2}{\sqrt{1 - w^2/c^2}}, \tag{3}$$

are then written down, m_0 being a constant.[2] Equations (2) are then taken as the equations of motion; equation (3) represents, moreover, the equality of rate of doing work and rate of increase of kinetic energy. The quantity $\mathbf{K}/k$ represents the force, and the generalizations of mass m and kinetic energy T are consequently

$$m = \frac{m_0}{\sqrt{1 - w^2/c^2}},$$

$$T = m_0c^2\left\{\frac{1}{\sqrt{1 - w^2/c^2}} - 1\right\},$$

where the -1 in the expression for T is introduced to provide for T being zero when $w = 0$. We thus have

$$T = c^2(m - m_0).$$

All that is contained in this derivation is the conclusion that the *change of* T is proportional to the *change of* m. The argument in no sense implies that m_0 represents the energy when the velocity is zero. Nor is it a quibble to deny it the privilege of this extension of meaning. So far, the energy of the particle when at rest has absolutely no status or meaning in the discussion. In order to go further, it is necessary to make use of postulates separate from the restricted theory of relativity, although, of course, not inconsistent with it. Even then, certain reservations are necessary in providing an interpreta-

[2] Or in a slightly more general form of the equations, an invariant which is not necessarily a constant.

tion to the meaning of expression (1). Such discussion of this matter as is usually made is made from the standpoint of the electromagnetic theory and, in particular, is made in terms of the ponderomotive forces $(\mathbf{E} + [\mathbf{uH}]/c)$ per unit charge, where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, respectively, and $\mathbf{u}$ refers to velocity. It is well known that, if $k \equiv (1 - u^2/c^2)^{-1/2}$, then $k(\mathbf{E} + [\mathbf{uH}]/c)$, $ik(\mathbf{E}\cdot\mathbf{u})/c$ constitute a 4-vector. Since many of the results which accrue owe their origin to the 4-vector nature of the ponderomotive force rather than to its electromagnetic nature, we shall, in what follows, consider a general 4-vector kF_x, kF_y, kF_z, $ik(\mathbf{F}\cdot\mathbf{u})/c$. This force 4-vector is, of course, perpendicular to the velocity 4-vector in the sense that the four-dimensional scalar product of the two 4-vectors is zero.

Now if ρ is a density, defined in terms of an invariant associated with an element of volume $d\tau$, as the invariant divided by $d\tau$, then the quantity $\rho d\tau$ is invariant. Thus, since dt/k is invariant, $F_x\rho d\tau dt$, $F_y\rho d\tau dt$, $F_z\rho d\tau dt$, $i(\mathbf{F}\cdot\mathbf{u})\rho d\tau dt/c$ is a 4-vector. Expressing the law of transformation of the first component of this 4-vector, we have:

$$F_x'\rho' d\tau' dt' = \beta F_x \rho d\tau dt - \frac{\beta v}{c^2}(\mathbf{F}\cdot\mathbf{u})\rho d\tau dt, \qquad (4)$$

where v is the velocity of the origin of the dashed system S' with respect to the origin of the undashed system S, and where $\beta = (1 - v^2/c^2)^{-1/2}$. The quantity $\mathbf{F}\rho d\tau$ will figure in the light of an external force on an element.

We shall suppose that in the system S' an entity, composed of the stuff whose density is ρ' is built up, from an initial condition at $t' = t_1'$ in which the elements of the entity are all at infinity.[3] The final condition of the entity in S' shall be one constant with t', and we shall suppose that $t' = t_2'$ is a time sufficiently late that this condition of constancy has been

[3] Stripped of generality, we propose to consider the construction of an electron by bringing up the elements of its charge from infinity in S'. Of course, the forces F must not be regarded as electrical in the ordinary sense, since to so regard them would involve recognizing that in the problem of the construction of an electron, for example, they would have, over a surface surrounding the final electron, a finite surface integral, such as to correspond to a charge equal and opposite to that of the electron.

well established. We shall fix our attention upon that particular process of constructing the entity in S' in which, at each instant,

$$\int\int\int \mathbf{F}'\rho' d\tau' = 0, \tag{5}$$

where the integral is taken at each instant at constant t'. Such a situation could have prevailed in the case of ordinary charges, had the charges been brought up slowly against their electrostatic repulsions.

We shall now examine the situation from the system S with respect to which the origin of S' has the velocity v, parallel to the axis of x. Let us integrate (4) first with respect to the time between the limits $t' = t_1'$ and $t' = t_2'$, which are constant throughout the system S', so that the corresponding limits for the time in S will be t_1 and t_2, such that

$$t_1 = \beta\left(t_1' + \frac{v}{c^2}x'\right); \qquad t_2 = \beta\left(t_2' + \frac{v}{c^2}x'\right).$$

If t_{01} and t_{02} are the corresponding time limits in S appropriate to the origin of S' at t_1' and t_2', respectively,

$$t_{01} = \beta t_1' \qquad \text{and} \qquad t_{02} = \beta t_2'.$$

Thus,

$$t_1 = t_{01} + \frac{\beta v}{c^2}x'; \qquad t_2 = t_{02} + \frac{\beta v}{c^2}x'. \tag{6}$$

Thus, integrating (4) in accordance with the above, with regard to time, and then integrating throughout space, we have:

$$\int\int\int d\tau \int_{t_{01}+\beta v x'/c^2}^{t_{02}+\beta v x'/c^2} F_x \rho dt - \frac{v}{c^2}\int\int\int d\tau \int_{t_{01}+\beta v x'/c^2}^{t_{02}+\beta v x'/c^2} (\mathbf{F}\cdot\mathbf{u})\rho dt = \frac{1}{\beta}\int\int\int d\tau' \int_{t_1'}^{t_2'} F_x'\rho' dt'. \tag{7}$$

Now, in accordance with the criterion (5), the right-hand integral in (7) is zero, so that

$$\int\int\int d\tau \int_{t_{01}+\beta v x'/c^2}^{t_{02}+\beta v x'/c^2} F_x \rho dt = \frac{v}{c^2}\int\int\int d\tau \int_{t_{01}+\beta v x'/c^2}^{t_{02}+\beta v x'/c^2} (\mathbf{F}\cdot\mathbf{u})\rho dt. \tag{8}$$

If we should ignore the dependence of the time limits upon x', the left-hand side, being the time integral of the external force in the direction of the final velocity, might be christened the momentum acquired by the system during the process of its construction as viewed in the system S. Also, the right-hand side would be, under the same conditions, v/c^2 times the time integral of the rate of doing work on the entity by the forces, or, in other words, v/c^2 times the total work done on the entity during its construction. We should thus have

$$\frac{\text{Momentum acquired}}{v} = \frac{\text{Total work performed in construction}}{c^2}. \quad (9)$$

If the left-hand side be christened the mass, we should have

$$\text{Mass of entity} = \frac{\text{Total work done in constructing entity}}{c^2}. \quad (10)$$

We shall call the value of the mass as determined in this way, m_1. However, the time limits in (7) do involve x', so that in order that the statements (9) and (10) should apply, it is necessary to form a new meaning for momentum acquired and work performed and speak not, for example, of the momentum acquired between two time limits t_{01} and t_{02}, but of the sum of the time integrals of the momentum acquired between time limits separated by $t_{02} - t_{01}$, but with the actual limits for each individual element as specified in (6) and (7). The time limits appropriate to each element are perfectly explicitly defined in terms of the conditions in the system S', which is perfectly definitely related to S through the velocity of transformation v.

However, apart from the foregoing difficulties and adjustments, it would yet remain to show that the left-hand integral in (8), which is concerned with the forces acting on the entity in its creation in final form, is the equivalent of the momentum of the entity of velocity v in the sense in which momentum has meaning in its actual use. The momentum of an entity as it occurs in dynamics makes its appearance through its time rate

of change when the entity is accelerated; and the story in this matter has no connection with the story concerned in the construction of the entity. The postulates concerned in it have to do with the finally constructed entity, and are entirely separate from the mechanism of construction. Thus, in electrodynamics, the expression for the mass and momentum of an electron arise from the postulate that the completed entity, the electron, when under the influence of external electromagnetic forces, moves in such a way that (5) continues to apply for the entity as viewed in S', even though it has an acceleration in that system, although, of course, zero velocity. This extension is a real extension, not involved in any way in our criterion (5) as already used. The final state in the construction of the electron is a steady one with no external applied force. The state contemplated in the accelerated motion of the electron is one where an external force exists and is balanced by a force originating in the electron and resulting from its acceleration. In order to avoid confusion, we shall change **F** to **G** in speaking of the forces on the entity regarded as completely formed, the forces concerned in its bodily motion with acceleration. Moreover, the forces **G** will, of necessity involve parts originating in the entity itself, in order that the criterion

$$\iiint_{t' = \text{const.}} \mathbf{G}'\rho' d\tau' = 0 \qquad (11)$$

shall hold.

It will be convenient to split **G** into two parts, a part $\mathbf{G}_i$, originating from the entity itself, and a part $\mathbf{G}_0$, originating from external causes. Then, (11) becomes

$$\iiint \mathbf{G}_0'\rho' d\tau' = -\iiint \mathbf{G}_i'\rho' d\tau'. \qquad (12)$$

Following the analogy of electrodynamics, it must now be our business to work out the right-hand side of (12) as a function of the motion. Then the coefficient of the acceleration will be the "rest mass" which we shall designate as m_{02}, to permit of a distinction from m_{01} which we shall regard as the value, for zero velocity, of the quantity m_1 already defined in another way. Viewed from the system S, the momentum then takes

the form [4] $m_{02}v/(1 - v^2/c^2)^{1/2}$, and the mass takes the form

$$m_2 = m_{02}/(1 - v^2/c^2)^{1/2}, \tag{13}$$

where, as already stated, m_{02} is the coefficient of the acceleration in the development of

$$\iiint \mathbf{G}_i'\rho' d\tau'.$$

However, we cannot develop this expression at all unless we make some postulate as to how $\mathbf{G}_i'$ depends upon ρ', and upon the acceleration, etc., in the system S'. This fact brings out very clearly the lack of any fundamental association between m_1 and m_2, apart from the particular postulates concerned with the motion. If we make $\mathbf{G}_i'$ depend upon the acceleration in the same way that the electrodynamic quantity $\mathbf{E}_i + [\mathbf{E}\mathbf{H}_i]/c$ depends upon the acceleration of the elements of charge, then the right-hand side works out, as is well known, to $(e^2/6\pi ac^2) \times$ acceleration, for a spherical shell of total charge e and radius a, so that m_{02} becomes for this case,[5]

$$m_{02} = e^2/6\pi ac^2. \tag{14}$$

On the other hand, it is possible to work out m_{01} without any appeal to the equation of motion of the entity as a whole. For expressing the transformation of the 4-vector $\mathbf{F}\rho d\tau dt$, $i(\mathbf{F}\cdot\mathbf{u})\rho d\tau dt/c$ in the manner reciprocal to that already used and integrating as before, we have:

$$\iiint d\tau \int_{t_{01}+\beta vx/c^2}^{t_{02}+\beta vx/c^2} F_x \rho dt = \beta \iiint d\tau' \int_{t_1'}^{t_2'} F_x'\rho' dt' + \frac{\beta v}{c^2} \iiint d\tau' \int_{t_1'}^{t_2'} (\mathbf{F}'\cdot\mathbf{u}')\rho' dt'. \tag{15}$$

[4] The quantity $k\frac{d}{dt}(k\mathbf{w})$, $k\frac{d}{dt}(kic)$ is a 4-vector, $\mathbf{w}$ being here the velocity of the entity as a whole, and k being $(1 - w^2/c^2)^{-1/2}$. Thus we have

$$m_{02}k\frac{d}{dt}(k\mathbf{w}) = m_0\beta\left\{k'\frac{d}{dt'}(k'\mathbf{w}') + \frac{v}{c}k'\frac{d}{dt'}(k'ic)\right\}.$$

Having regard to the fact that, in S', $k' = 1$, also $\frac{dk'}{dt} = 0$, and $k = \beta$, we have

$$\frac{d}{dt}\frac{m_{02}\mathbf{w}}{(1 - w^2/c^2)^{1/2}} = m_{02}\frac{d\mathbf{w}'}{dt'}$$

which establishes the momentum as $m_{02}\mathbf{w}/(1 - w^2/c^2)^{1/2}$.

[5] We use Heavisidean units.

Now the first integral on the right is zero on account of (5). Thus, we have $m_1 v$ equal to the second integral on the right-hand side; and consequently

$$m_1 = \frac{\beta}{c^2}\int\int\int d\tau' \int_{t_1'}^{t_2'} (\mathbf{F}' \cdot \mathbf{u}')\rho' dt'. \tag{16}$$

The integral in (16) represents the time integral of the rate of doing work by the external forces in the complete construction of the entity. In other words, it represents the work performed in constructing the entity. Again we cannot evaluate it without some postulate as to the form of $\mathbf{F}'$, but now no question of the dependence of $\mathbf{F}'$ upon the acceleration of the entity as a whole is involved. If we assume that the entity is constructed in S' in the manner already stated, i.e., in opposition to static forces, and if we further assume that those forces are the forces of electrodynamics, so that $\mathbf{F}'$ is everywhere equal to the negative of the electrostatic force which we should find in the ordinary way for the distribution ρ', and, finally, if we consider the limiting problem of a spherical shell of radius a and total charge e, we find, for the integral, the value $e^2/8\pi a$, so that

$$m_1 = \frac{e^2}{8\pi a c^2}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{17}$$

and

$$m_{01} = \frac{e^2}{8\pi a c^2}. \tag{18}$$

There is thus a discrepancy between m_{01} and m_{02}, amounting to $e^2/24\pi a c^2$.[6]

It would, of course, be possible to remove this discrepancy by assuming that the forces designated by $\mathbf{G}$ differed from those calculable from classical electrodynamics in such a manner as to change the constant in the expression for the force as a function of the acceleration. A less drastic departure from the classical standpoint is to make that departure in respect

[6] One is, of course, continually conscious of a connection between this discrepancy and the corresponding discrepancy involved in the old problem of the Lorentz electron and the conservation of energy, which has been discussed by Abraham and Poincaré. For a further discussion of this matter, see a paper by the present writer, "Classical Electrodynamics and the Conservation of Energy," JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 212, p. 563, November, 1931.

of the equation of motion only, and instead of assuming (12), assume

$$\int\int\int \mathbf{G}_0'\rho' d\tau' = -\frac{6}{8}\int\int\int \mathbf{G}_i'\rho' d\tau'. \qquad (19)$$

Such an equation has perfectly definite meaning. Moreover, it leaves the circuital relations of electrodynamics untouched. The 4-vector character of $\mathbf{G}_0$ and $\mathbf{G}_i$ also continues to insure conformity with the theory of relativity. The law denies (11), but it does not affect (5) in any way, which latter equation is concerned only with the construction of the entity and has no relation to any law concerned with its bodily motion as a whole.

ANOTHER FORMULATION OF THE PROBLEM.

Let us return once more to (7), but let us modify it so that the time limits are constant values of t rather than of t'.

$$\int\int\int d\tau \int_{t_1}^{t_2} F_x\rho dt - \frac{v}{c^2}\int\int\int d\tau \int_{t_1}^{t_2} (\mathbf{F}\cdot\mathbf{u})\rho dt = \frac{1}{\beta}\int\int\int d\tau' \int_{t_1' = t_1/\beta - vx'/c^2}^{t_2' = t_2/\beta - vx'/c^2} F_x'\rho' dt'. \qquad (20)$$

The left-hand side is no longer zero, because since the limits of the right-hand integral involve x', that integral is not zero. If, as is natural, we call the first integral, on the left, the total momentum acquired, and the second integral, the total work performed by the external forces in constructing the entity, we no longer have the relation represented by (9). In order to evaluate the right-hand side, we have again to make some postulate as regards F_x'. If we confine our interest to the spherical shell of radius a, with total charge e, as before, and if, as before, we take for F_x' the negative of the electrostatic forces which would be produced by this system we can evaluate the right-hand side. We first observe that since F_x' is zero everywhere in the neighborhood of the lower limit, we may discard the term vx'/c^2 there. The integral may then be written

$$\int\int\int d\tau' \int_{t_1/\beta}^{t_2/\beta} F_x'\rho' dt' + \int\int\int d\tau' \int_{t_1/\beta}^{t_2/\beta - vx'/c^2} F_x'\rho' dt'.$$

The first integral of this expression is zero in accordance with (5), since it is evaluated between constant limits of t'. As regards the second integral, we have the fact that, in the neighborhood of the upper limit, F_x' and ρ' have become constant with the time. The integral in question thus becomes

$$-\frac{v}{c^2}\int\int\int x' F_x' \rho' d\tau'.$$

If we put in the negative of the electrostatic force for F_x' at a point, this expression can easily be evaluated as

$$\frac{e^2 v}{24\pi a c^2}.$$

Equation (20) thus becomes

$$\int\int\int d\tau \int_{t_1}^{t_2} F_x \rho dt - \frac{v}{c^2}\int\int\int d\tau \int_{t_1}^{t_2} (\mathbf{F}\cdot\mathbf{u})\rho dt = \frac{e^2 v}{24\pi a c^2 \beta} \quad (21)$$

and (9), of the older formulation becomes replaced by

$$\frac{\text{Momentum acquired}}{v} = \frac{\text{Total work performed in construction}}{c^2} + \frac{e^2}{24\pi a c^2 \beta}.$$

The mass energy relation thus becomes destroyed, and to an extent comparable with the whole value of the two quantities whose equality is desired. On the other hand, as a compensation for this defect, the first term in (21) now agrees with the momentum arrived at from classical electrodynamics. For, let us once more write down (15), but modified to the case where the time limits are constant values of t rather than t' We then have

$$\int\int\int d\tau \int_{t_1}^{t_2} F_x \rho dt = \beta\int\int\int d\tau' \int_{t_1' = t_1/\beta - vx'/c^2}^{t_2' = t_2/\beta - vx'/c^2} F_x' \rho' dt' + \frac{\beta v}{c^2}\int\int\int d\tau' \int_{t_1' = t_1/\beta - vx'/c^2}^{t_2' = t_2/\beta - vx'/c^2} (\mathbf{F}'\cdot\mathbf{u}')\rho' dt'. \quad (22)$$

As we have already remarked, the occurrence of x' in the lower limits may be discarded. Moreover, for the electromagnetic

case, the first integral on the left-hand side has already been evaluated for the spherical electron of charge e and radius a, and it has been found to have the value $e^2v/24\pi ac^2$. In the second integral on the right-hand side, the occurrence of x' in both limits may be discarded, since $(\mathbf{F}' \cdot \mathbf{u}')$ is zero when the situation has become steady. The integral thus reverts to an integral between constant time limits, and it is in fact the integral concerned in (16) which has been stated to have the value $e^2/8\pi a$.

Thus (22) becomes

$$\int\int\int d\tau \int_{t_1}^{t_2} F_x \rho dt = \beta \left\{ \frac{e^2 v}{24\pi ac^2} + \frac{e^2 v}{8\pi ac^2} \right\} = \frac{e^2 v}{6\pi ac^2} \beta, \quad (23)$$

which gives a mass $e^2\beta/6\pi ac^2$, in agreement with that calculated from the equation of electronic motion of classical electrodynamics.

On the other hand, in the mass-energy relation, we have the discrepancy represented by the term on the right-hand side of (21). It may be of interest to express, in a different form, the extent of the discrepancy.

Calling $\int\int\int d\tau \int_{t_1}^{t_2} F_x \rho dt$ the momentum M acquired by the entity in construction in the system S, we have seen in (23) that

$$M = \frac{e^2 v \beta}{6\pi ac^2}.$$

Thus, calling W the work performed in constructing the entity in system S, we have from (21),

$$M - \frac{v}{c^2} W = \frac{6M}{24\beta^2},$$

$$M \left(1 - \frac{1}{4\beta^2} \right) = \frac{v}{c^2} W,$$

so that the relation is

$$\text{Mass} = \frac{W}{c^2} \left(1 - \frac{1}{4\beta^2} \right)^{-1}.$$

The statement

$$\text{Mass} = W/c^2$$

is true as regards order of magnitude with a maximum error of 25 per cent.

It is not readily possible to make a reasonable adjustment of the postulates to remove the discrepancy, for its removal necessitates providing for a zero value for the right-hand side of (20), and apart from trivial exceptions this quantity cannot be zero. In fact, even if one should split the integral into two parts, one involving only fixed time limits and the other, the variable parts of the time limits, and if we should give up the postulate (5), the second part, depending as it does upon v, could only be made to annul the first part for a critical value of v.

In conclusion, it may be stated that the object of this paper is not to attempt a complete straightening out of the problem so as to provide for an exact relation of the form mass = energy/c^2, but rather to air the situation as it stands in the light of the classical theories of the subject.

BARTOL RESEARCH FOUNDATION OF
THE FRANKLIN INSTITUTE,
July 26, 1931.

THE RELATIVE PERMEABILITY OF IRON, NICKEL AND PERMALLOY IN HIGH FREQUENCY ELECTROMAGNETIC FIELDS.

BY

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University of Wisconsin, Madison.

ABSTRACT.

The relative magnetic permeability of iron, nickel and permalloy was measured over the range of wave-lengths from 70 to 200 meters by the heterodyne balance method and by the resonance method. Nothing suggestive of the anomalous dispersion of the magnetic permeability reported in this region by Wwedensky and Theodortschik was found when certain precautions were observed in carrying out the measurements. When these precautions were neglected irregular breaks resulted in the curves of permeability as a function of wave-length which varied in position with the conditions of the circuit.

The high frequency resistance of iron, nickel and permalloy wires was measured over the range of approximately fifty thousand kilocycles from two to three meter wave-lengths. What appeared to be anomalous variations at certain frequencies were found not to exist when the proper precautions were observed in making the measurements.

SIGNIFICANCE OF ANOMALOUS PERMEABILITY AT A CRITICAL FREQUENCY.

Arkadiew [1] argues that since the magnetic permeability of ferromagnetic materials is of the order of hundreds or thousands of units in slowly changing magnetic fields and unity for all materials in the magnetic fields of light and infra-red waves, there must somewhere be an intermediate region of magnetic absorption characterized by large variations in permeability. Such anomalous values he interprets as resonance phenomena analogous to the electrical absorption of electromagnetic waves at wave-lengths corresponding to the natural frequency of the electric centers in dielectrics. He thus believes that the magnetic vector of a wave of suitable frequency should produce an effect in ferromagnetic media analogous to that produced by the electric vector in dielectrict

[1] Arkadiew, W., *Phil. Mag.* (6), 50: 157–163. 1925.

and that consequently the permeability should behave as critical frequencies in a manner corresponding to the behavior of the dielectric constant. He assumes that the elementary magnets rotate in fields under the same laws of viscous elastic motion generally applied to the displacement of electric centers in dielectrics. Of the experimental results shown by Arkadiew in a composite curve of the results of several investigators, the measurements of Wwedensky and Theodortschik [2] at wave-lengths extending from sixty-four to five hundred meters show what appears to be an effect most suggestive of magnetic absorption. A maximum value of the permeability appears at a wave-length of a hundred meters, between minima at ninety and one hundred and twenty meters.

PURPOSE OF THE PRESENT WORK.

In view of the theoretical significance of an anomalous permeability at a critical frequency, the first object of the present work was the verification of the anomalous relative permeability reported for iron and nickel by Wwedensky and Theodortschik at wave-lengths in the neighborhood of one hundred meters. It was then proposed to study the effect of variations in field strength, and changes in the crystalline state of the material under investigation upon the value of the anomalous permeability in an attempt to gain some insight as to the identity of the elementary magnetic units and the nature of the constraints to which they are subjected. Since permalloy shows most ferromagnetic phenomena in a greatly enhanced degree, the third and major objective of the work was the investigation of the relative permeability of permalloy over a range of frequencies including those at which anomalies had been reported in its two constituents, iron and nickel.

RESULTS OF OTHER INVESTIGATORS.

Laville [3] published results which indicate that the magnetic permeability of iron wires undergoes no sudden or irregular change in magnitude over a range of wave-lengths extending from three to eighty-five and one half meters. While

[2] Wwedensky, B., and Theodortschik, K., *Ann. d. Physik* (4), **68**: 463–480. 1922.

[3] Laville, G., *Ann. d. Physik.* (10), **2**: 328–404 (1924).

Laville's measurements did not include values of the relative permeability at wave-lengths near to and including one hundred meters, where Wwedensky and Theodortschik reported a maximum variation, the results were, nevertheless, contradictory in the band of wave-lengths from fifty-four to eighty meters where the results of the two observers overlapped.

In 1927 Wait [4] reported no anomalous relative permeabilities anywhere in the range of wave-lengths from eighty to seventeen hundred meters for either iron wires, iron powders, or powdered magnetite.

In the same year Mitiaev [5] investigated the influence of a constant magnetic field on the magnetic spectra observed by Wwedensky and Theodortschik. Mitiaev confirmed the dispersion bands reported by Wwedensky and Theodortschik and found their position to remain unaltered.

In 1927 Gutton and Mihul [6] reported a confirmation of the results of Laville which means that their results disagreed with those of Wwedensky and Theodortschik in that they found no anomalous changes in permeability at or near wave-lengths of one hundred meters.

In the latter part of 1927 Wait [7] and two co-workers from the Bureau of Standards, measured the high frequency resistance of iron wires in the region of wave-lengths from seventy-nine to one hundred and twenty meters. The anomalous resistance reported by Mitiaev as a confirmation of some of Wwedensky and Theodortschik's work was not found. Wait likewise reports that he repeated his former permeability measurements with a sample of iron wire in which Wwedensky and Theodortschik had observed selective magnetic absorption and that he (Wait) again got negative results.

METHODS OF MEASUREMENT.

In the present study of the magnetic permeability of iron, nickel, and permalloy in high frequency fields, three inde-

[4] Wait, G. R., *Phys. Rev.* (2) **29**: 566–578 (1927).

[5] Mitiaev, W. K., *Zeits. f. Physik.*, **38**: 716–726 (1926).

[6] Gutton, C., and Mihul, I., *Comptes rendus*, **184**: 1234–1237 (1927).

[7] Wait, G. R., Brickwedde, F. G., and Hall, E. L., *Phys. Rev.* (2), **32**: 967–973 (1928).

pendent methods of measurement were employed. Both the first and second method are based upon the fact that when a cylindrical ferromagnetic specimen is placed inside of a solenoid the change in self-inductance which results may, under the proper circumstance, be taken as a measure of the magnetic permeability of the specimen. The two independent methods employed differ in the manner in which this change in self-inductance is measured.

In the so-called "beat note" method, a sensitive heterodyne balance was used to detect and measure the changes in self-inductance. The change in capacity necessary to balance out the beat note between two oscillators, which were tuned to the same frequency before the introduction of the specimen, is related to the permeability of the specimen by the equation of Belz.[8]

$$\mu = 1 + \frac{V}{V'}\frac{\Delta C}{C}.$$

Where V and V' signify the volume of the specimen and specimen chamber respectively, C is the total capacity of the oscillating circuit and ΔC the change in capacity necessary to eliminate the beat note.

In the resonance method the permeability of the specimen is measured in terms of the change in the capacity of a simple receiver circuit necessary to restore it to resonance with a loosely coupled oscillator. The equation expressing the desired relationship follows simply from the general Belz formula and is

$$\mu = \frac{V}{V'}\frac{C}{C'}.$$

C is the capacity of the receiver circuit at resonance without the magnetic specimen and C' is the capacity when this same circuit is retuned to resonance with the specimen in place.

In the third method of measurement used at wave-lengths in the neighborhood of two meters, the resistance of the specimen was determined by including it as part of the conducting circuit of a rectangular loop of known capacity, inductance and resistance. The loop, arranged as a Lecher

[8] Belz, M. H., *Phil. Mag.* (6), **44**: 479–501 (1922).

wire system, was tuned to resonance with a loosely coupled oscillator, the magnitude of the high frequency current was noted and the magnetic sample was then removed from the circuit and replaced by copper wire of the same dimensions. The change in the resistance of the circuit necessary to again produce the same current at resonance constituted a measure of the high frequency resistance of the specimen which is related to the permeability by the formula of Stephen as used by Gutton and Mihul.

$$R = R_0[a\pi\sqrt{\mu cn} + \tfrac{1}{4}].$$

PREPARATION AND CHARACTERISTICS OF THE SPECIMENS.

The specimens of ferromagnetic material used in the relative permeability measurements by methods one and three consisted of fine iron wires, and nickel wires rolled into thin ribbons, each mounted in a glass tube, care being taken to avoid metallic contact. The specimens were electrically annealed after they had been mounted to keep the different strands as nearly uniform magnetically as possible. When more than a few strands were mounted they were mutually insolated by protective coatings of celoydin.

In addition to wire filaments, the specimens used in method two consisted of fine metallic iron powder and permalloy powder packed and sealed into glass tubes. Each tube made a slip fit with the specimen chamber and was provided with a stop to insure equal depths of insertion on subsequent runs.

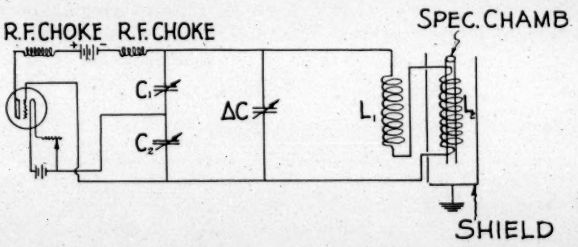
APPARATUS.

The final set up used for measurements by method one included two oscillators of the Colpitts type. These were loosely coupled to an aperiodic intermediate circuit the terminals of which were connected to the first grid and negative filament lead of an amplifier of standard form. Telephones were included in the plate circuit of the last stage of the amplifier. In oscillator No. 1., Fig. 1., a 201A General Electric tube was used, operating on a six volt storage battery in the filament circuit, and a ninety volt B battery in the plate circuit. The inductance L was a coil wound in a single layer on glass tubing, the number of turns and cross sectional area

of both coil and wire varying for different ranges of frequency over which measurements were attempted. In the first experiments Cardwell twenty-three plate condensers were used at C and C' while at ΔC a one half plate General Radio condenser in parallel with C served as a vernier. Since it took a turn of over one hundred divisions on the scale attached to the vernier condenser to equal a turn of one division on the Cardwell condenser adjustment of capacity by very finely graded steps was possible.

Each oscillator was enclosed in a separate metal lined box with its A and B batteries. Each of the five-turn pick-up coils of the intermediate circuit was enclosed in the same shielded box as the oscillator to which it was coupled. The

FIG. 1.



leads from these intermediate circuit coils were fixed, passing through grounded brass tubing from which they were insulated by thick rubber, and the coupling with either oscillator was adjusted by variation of the angle which the intermediate circuit coil made with the respective oscillator coil.

Oscillator No. 2 was made up with a five watt General Electric tube using an eight-volt storage battery in the filament circuit and a ninety-volt B battery in the plate circuit. No vernier condenser was used, nor was there any shielded specimen chamber.

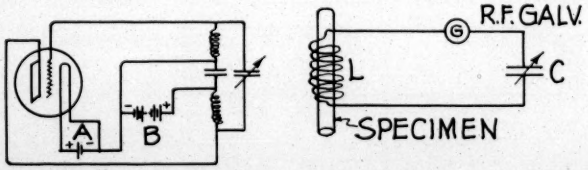
The tube used in the receiving amplifier was a Wd 12 which, along with its A and B battery, was located inside of a separate metal lined and grounded box as in the case of the

two oscillators. The telephones were standard Western Electric 2000-ohm radio receivers.

An electric tuning fork served as a standard of frequency with which to match the beat note between the two high-frequency oscillators by the adjustment of the vernier condenser. When no secondary beats were heard between the two audio-frequency notes, it was definitely known that the beat note was of identical frequency with the tuning fork.

The method of connecting the apparatus for method two is shown in Fig. 2. A single Precision condenser C, provided with a slow motion screw was used both for obtaining successive values C and ΔC. By means of the slow motion screw on this condenser the capacity of the circuit could be varied from 52 to 1190 m.m.f. by measured steps of .5 m.m.f. To get rid of the capacity effect of the operator's hand the slow motion screw was turned by a bakelite rod.

FIG. 2.



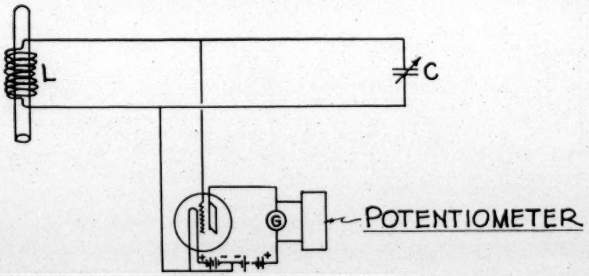
The specimen coils finally adopted were made of No. 14 cotton covered wire wound in shellac on hard wood cylinders 5.9 centimeters in diameter. The number of turns varied with the range of wave-lengths to be covered by the resonant circuit. A hole bored out along the axis of each cylinder to make a slip fit with the glass specimen tube insured identical centering of the specimen for successive insertions.

The radio-frequency galvanometer used to detect maximum current at resonance had a resistance of 4.5 ohms and required 115 milliamperes for full scale deflection. The leads from condenser to specimen coil, through the galvanometer, were rigid brass rods supported at one end by large binding posts on the Precision condenser, and by a bakelite plate at the other, through which they extended far enough to make

soldered joints to the ends of the specimen coil. The oscillator used was a split-circuit Hartley, Fig. 2, which was found to be the least susceptible to disturbance of any of the circuits tested.

It is well known that a circuit can be tuned to sharp resonance only when it possesses low resistance. In the circuit just described the specimen coil and leads were of relatively large cross section and were joined by well soldered connections so that the principal resistance in the circuit was that of the galvanometer. Clearly, the particular circuit used admitted of little improvement in this respect, because reduction of the resistance of the galvanometer meant reduced sensitivity. Consequently a vacuum tube circuit was employed which permitted the use of a sensitive high resistance galvanometer without destroying sharpness of resonance. Fig. 3 shows the arrangement of instruments for this circuit.

FIG. 3.

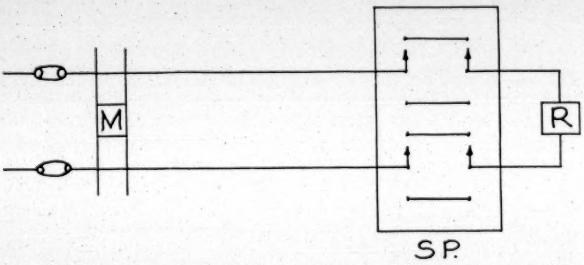


It will be seen from the figure that the receiver consisted of the precision condenser joined directly to the specimen coil by the brass rod leads, thus forming a simple parallel resonance circuit of very low resistance. The two pick-off wires, as shown in the figure, were connected to the grid and filament of a vacuum tube which is essentially a potential actuated instrument. Thus it takes no appreciable current from the pick-up circuit and therefore does not appreciably affect the desired low resistance. The galvanometer used in the plate circuit of this tube was a Weston D.C. instrument with a

resistance of 161.1 ohms and a sensitivity of $.25 \times 10^{-6}$ amperes per division. Some form of potentiometer must of course be included to counteract the plate voltage of the tube and prevent it from burning out the galvanometer.

The circuit used in method three is shown in Fig. 4. The sliding meter *M* was a radio-frequency thermo-galvanometer with a current-squared scale, requiring 115 milliamperes for full scale deflection. It was connected to the wires of the rectangular loop by square aluminum plates pierced near their centers and serving as sliding contacts for the meter, and, at

FIG. 4.

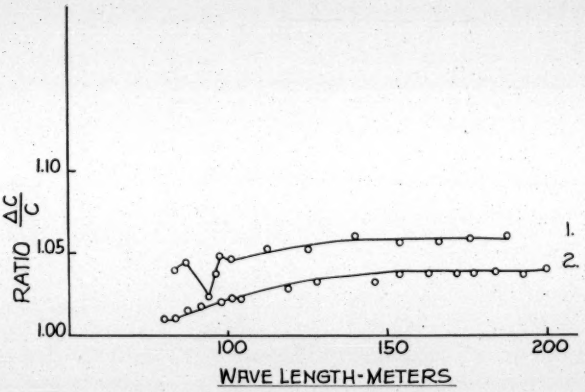


the same time, as shields to prevent spurious reaction from the dead end of the line. The specimen panel *SP* enabled the operator to switch, at will, from the wires under test to standard wires. A constant inductance type rheostat at *R* was used to vary the resistance of the tuned loop without producing a detectable change in its inductance or capacity. The oscillator used to generate the two meter waves was of the standard Ultraudion type used at very high frequencies. Four inch No. 14 copper wires soldered to the grid and plate leads of a 5 watt tungsten filament tube lead to the terminals of a .005 m.f. mica condenser. This comprised the fly wheel circuit to which the usual B battery and A battery leads were connected through radio-frequency chokes. Tuning was effected by a one half plate General Radio variable condenser shunted across the fly wheel circuit at the grid-plate side of the circuit.

RESULTS.

While first experiments with the heterodyne balance showed what appeared to be anomalous variations in permeability with frequency of the order of magnitude anticipated, these apparent variations disappeared almost entirely when precautions were taken to keep the oscillator current constant over the entire frequency range. There remained, however, much smaller variations which were shown by repeated runs to be definitely associated with, and to reach a maximum at, a particular frequency. Curve No. 1, Fig. 5, is typical of what

FIG. 5.

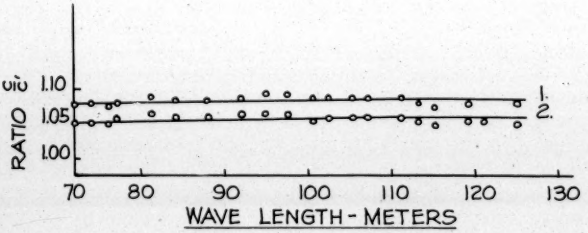


was found. It shows a well defined minumum at 94 meters between maxima at 88 and 97 meters. Subsequent experiments with the heterodyne oscillator circuit showed, however, that this apparent anomaly, also, was due to circuit conditions and was in no way characteristic of the magnetic material of the specimen. Briefly, it was found that curves of the type of No. 1 were obtained when the magnetic specimen was placed in the oscillating circuit in such a manner as to change, not only the self-inductance of the oscillator, but also, the mutual inductance between the plate and grid circuit. Thus it was found that the anomaly was characteristic of the sensitivity of the oscillator to changes in "feed back" at different

frequencies. This was conclusively proved by the fact that when a separate and properly shielded auxiliary specimen coil was used in conjunction with the main inductance so that the introduction of the specimen did not alter the "feed back" ratio, results were obtained as in curve No. 2, Fig. 5. As will be seen there is a conspicuous absence of anything suggestive of selective absorption.

Results obtained by the resonance method at wavelengths from 70 to 120 meters were very similar in that what at first appeared to be variations in magnetic permeability at definite frequencies was finally traced to random variations caused by too broad tuning, circuit perculiarities due to the presence of a metallic shield within the specimen coil, and variations in coupling between the oscillator and resonance circuit. When all of these difficulties had been identified and eliminated, the measured values of the permeability were without variation within the limits of error, some two or three percent. This is shown in curve No. 1 for Western Electric iron powder and in curve No. 2 for powdered permalloy (Fig. 6). In as much as the variations looked for

FIG. 6.

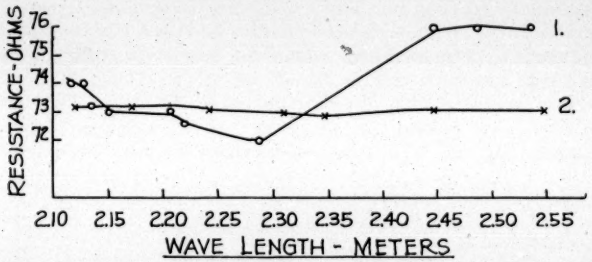


were of the order of 60 or 70 per cent., it is believed that they were not missed through any lack of sensitivity.

Measurements of the high-frequency resistance of iron, nickel and permalloy wires in the neighborhood of two meters by method three, like those of the permeability in the 100 meter region, showed variations with frequency in the early experiments only. They became regular, however, and gave no suggestion of any sort of selective absorption when great

pains were taken to vary the frequency of the circuit without changing the ratio of inductance to capacity. When readings were taken for No. 36 iron and No. 38 permalloy wires over a range of frequencies obtained by varying the capacity alone of the loosely coupled driving oscillator, the results were as shown in curve No. 1, Fig. 7. When the tuning of the oscillator was effected by changing both the inductance and capacity simultaneously in amounts which kept their ratio to each other constant, the values obtained for the high-frequency resistance, which is a known function of the permeability, exhibited no anomalous variations but were as shown

FIG. 7.



in curve No. 2, Fig. 7. The connection between the readings and the inductance-capacity ratio of the oscillator is not clear other than the possibility that the effect was connected with the energy output, the oscillator oscillating harder with certain ratios of inductance to capacity than with others. Before this could be entirely established however, the two meter oscillator which had worked well for nearly a year broke down and as five watt tubes of the type are no longer made the experiments were terminated for the present at least.

CONCLUSIONS FROM THE PRESENT EXPERIMENTS AND COMPARISON WITH THE WORK OF OTHERS.

The final conclusion, based upon the experimental measurements described in this paper, is that there is no anomalous variation in the relative magnetic permeability of iron, nickel,

and permalloy at frequencies corresponding to the band of wave-lengths from seventy to two hundred meters, and, in particular, that there is no sharp break or phenomenon suggestive of electromagnetic absorption in the curves for these materials at or near one hundred meters. If such an effect does exist it must certainly be of the order of a one per cent. variation or less, and must therefore fall below the limits of experimental error which have been achieved with any form of apparatus as yet devised for this type ofmeasurements.

It is further concluded that there exists no irregular variation in high-frequency resistance, and consequently no corresponding change in magnetic permeability for iron and permalloy wires at wave-lengths in the neighborhood of two meters.

The results of the present experiments on iron and nickel thus confirm the work of Wait, Laville, and Gutton and Mihul and are contrary to that of Wwedensky and Theodortschik and Mitiaev abroad, and that of Kralovec in this country. In addition, the measurements on permalloy indicate a uniform trend in the relative permeability of this ferro-nickel alloy with no values giving sharp breaks suggestive of absorption.

By way of conclusion it is thought worth while to summarize, briefly, the difference between the experimental procedure of those investigators who find anomalous variations in relative permeability with frequency, and those who do not.

1. Those investigators who report an anomalous permeability from results obtained by the heterodyne balance method introduced their specimen in such a manner that they changed both the self-inductance of the specimen coil, as intended, and at the same time changed the mutual inductance between the grid and plate circuit thus altering the feed-back of energy with a consequent change in circuit conditions.

In the present work, precautions were taken, as previously described, to avoid this possible source of error, and no anomalous values of relative permeability appeared in the results. On the other hand, when these precautions were purposely neglected, in certain test runs, apparent anomalies resulted.

2. Those investigators who report anomalous relative permeabilities from results obtained by the resonance method

worked, in certain cases, with a type of resonant circuit which was abandoned in the present experiment because of lack of sharpness of resonance. In other cases, they worked with a specimen coil and specimen of such relative dimensions as to necessitate the use of an internal metal shield of uncertain effect. Furthermore, the strength of the current in the specimen coil was kept constant, in certain of the measurements in which anomalies are reported, by varying the coupling between oscillator and resonant receiving circuit.

In the present work with the resonance method, these conditions were eliminated, and no anomalous permeabilities were observed in the measurements, although apparent anomalies appeared in test runs with internal metallic shields, with circuits in which there was broad tuning due to galvanometer resistance, and in certain cases where the coupling was varied between driving and receiving circuit.

It is desired to emphasize the fact that the absence of these critical precautions in the measurements of those investigators who report anomalous values of relative permeability at certain frequencies, may well account for the discrepancy which exists in the results obtained by different observers in this field.

The writer wishes to express his sincere gratitude to the late Professor E. M. Terry, at whose suggestion the work was undertaken, for his unfailing interest and helpful advice throughout its progress. It is also desired to thank Professor H. B. Wahlin for his much appreciated criticism of the paper.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

A CERAMIC MATERIAL OF LOW THERMAL EXPANSIVITY.

In Volume 10 (June, 1929) and Volume 11 (February, 1930) of the *Ber. der. Deut. Ker. Gesell.*, W. M. Cohn presents the results of thermal expansion determinations on a ceramic body having the remarkably low coefficient of thermal expansion of 0.53×10^{-6} (106 parts per million) in the temperature range 0–200° C. His results indicate a slight contraction reaching a maximum of about ten parts per million at approximately 70° C., after which the material expands at a gradually increasing rate. A typical body composition was given as 43 per cent. steatite, 35 per cent. Zinzendorfer clay, and 22 per cent. anhydrous Al_2O_3.

To duplicate these interesting results a body was prepared of 43 per cent. talc, 35 per cent. kaolin from Florida, and 22 per cent. corundum, and subjected to a number of different heat treatments. Not only were expansions obtained similar to those found by Cohn, but it was possible to establish the fact that the low expansivity is a characteristic of one form of the mineral cordierite ($2MgO.2Al_2O_3.5SiO_2$). The formation of this compound apparently proceeds very slowly at 1,350° C. and with increasing rapidity at higher temperatures up to approximately 1,450° where dissociation begins. Contraction below 100° C. was noted only for certain specimens made with talc as the source of the MgO. The smallest length changes noted were those of the talc body heated for three days at 1,350° C. and were as follows (given in parts per million): 60°—contraction of 15 parts; 203°—expansion of 63 parts; 402°—expansion of 301 parts. Specimens made of reagent quality materials in the cordierite ratio, heated 20 minutes at 1,415° C. and containing approximately 90 per cent. of the ternary compound, showed the following expansions: 60°—15 parts; 140°—80 parts; 200°—160 parts. The same specimens, if held for some hours at 950° C., apparently

* Communicated by the Director.

invert to the form described by Rankin and Merwin as unstable (*A. J. Sci.*, **195**, 45, 1918) and the thermal expansion has been increased to 415 parts per million at 200° C. However, reheating at 1,350–1,400° C. causes reversion to the low expanding form.

The magnesium silicates, forsterite and clinoenstatite, undergo very large length changes when heated, the former being higher than that of corundum, and these are in turn exceeded by the expansion of $\propto CaO.SiO_2$ which was 2,020 parts per million at 200° C. The expansion of ortho zinc silicate is surprisingly low, 389 parts at 295°, and preliminary tests of natural beryl indicate that this material may have a thermal dilation lower than that of cordierite.

COMPOSITION AND THERMAL EXPANSION OF SOME SODA-LIME-SILICA GLASSES.

The thermal expansions of 13 soda-silica and 20 soda-lime-silica glasses were measured between room temperature and softening point by the interferometer method. The silica content of these glasses varied from about 52 to 80 per cent., the balance being either soda or soda and lime, but the lime did not exceed 16 per cent.

The relation between composition and expansion can be fairly accurately expressed by the following equations:

$$Tb_1 = 495 + 0.95B - 0.058B^2$$
$$Tb_2 = 535 - 0.045B^2$$
$$Eb_1 = 5 + 2.65B - 0.0200B^2$$
$$Eb_2 = 6 + 3.13B - 0.0275B^2$$
$$Tc_1 = 485 + 0.95(B + C) - 0.058(B + C)^2 + 216R$$
$$Tc_2 = 520 - 0.045(B + C)^2 + 238R$$
$$Ec_1 = 2.5 + 2.65B + 2.37C - 0.0200(B + C)^2$$
$$Ec_2 = 14 + 2.50B + 2.37C - 0.0200(B + C)^2$$

The "key" for these equations follows: T = temperature (° C.); E = expansion (microns per centimeter); B = percentage of soda; C = percentage of lime; R = ratio of the lime to the soda plus lime $\left(\frac{C}{B + C}\right)$; b indicates soda-silica

glasses; c indicates soda-lime-silica glasses; the subscript (1) indicates critical temperature or the expansion at that temperature, and subscript (2) indicates the softening temperature or the expansion at that temperature.

From the above equations it is thought one can obtain a fairly accurate idea of the temperature at which rapid expansion of any glass of the series studied begins, the temperature at which the glass begins to soften appreciably, and also the total expansions from room temperature to these temperatures. Then with an idea as to the general shape of thermal expansion curves for glass one can draw a fairly good graph to represent the thermal expansion of the glass. The equations also give some indication as to the range in which such glasses could be annealed because the annealing range is usually regarded as that temperature range between the critical temperature and the initial softening point.

ULTRA VIOLET AGRICULTURAL GLASS.

In view of the inquiries received at the bureau as a result of the increased sales promotion of "Agricultural Window Glass," "Ultra Violet Agricultural Glass," etc., for use in hen houses and other animal houses, the following information may be of interest:

The term "agricultural glass" is a misnomer that is subject to misunderstanding and open to misrepresentation of the product.

As presented to this bureau it represents a window glass made especially for transmitting ultra violet solar rays of wave-lengths 302 millimicrons and shorter. It is in reality a grade C or very poor grade B of this special glass, that contains optical flaws which make the glass unsightly in dwellings and solariums but which has practically the same ultra violet transparency as grades A and B.

It is well known that the ultra violet transparency of the different melts of these special window glasses is variable. Hence it is possible for "agricultural glass" to become the dumping ground for the poorer melts of ultra violet transmitting glass.

It is interesting to note that much of the present day sales promotion of these special window glasses for use for

healing human beings, is based upon experiments originally performed upon chicks, white rats and other animals. Attention is therefore called to a report to the council of physical therapy of the American Medical Association (*J.A.M.A.*, **88**, 1,562; May 14, 1927) showing that a transmission of 30 per cent. at 302 millimicrons is required in order to insure that the amount of ultra violet passing through the glass will be effective. Glass that transmitted only 5 per cent. at 302 millimicrons gave no protection from rickets and in this particular test even the 30 per cent. transmission did not effect a 100 per cent. cure of rickets.

SOUNDPROOF PARTITIONS.

The acoustic laboratory of the bureau has recently made sound transmission measurements on a set of wall panels the results of which should be of general interest.

The panels were constructed of cinder block and clay tile, and were built with instructions to use workmanship such as would ordinarily be used in a construction job. A description of the panels follows:

No. 1: Standard Straub hollow cinder block partition tile 4 by 8 by 18 inches. Weight of finished panel 29.7 lbs./ft.^{2} Thickness of wall 4 inches.

No. 2: New Jersey porous hollow clay partition tile, 4 by 12 by 12 inches. Weight of finished panel 27.5 lbs./ft.^{2} Thickness of wall 4 inches.

No. 3: New Jersey hollow clay tile 4 by 12 by 12 inches with 1-inch shells. Weight of finished panel 37.5 lbs./ft.^{2} Thickness of wall 4 inches.

No. 4: New Jersey standard clay partition tile 4 by 12 by 12 inches. Weight of finished panel 33.4 lbs./ft.^{2} Thickness of wall 4 inches.

In each case the panels were laid up in a mortar composed of one part Portland cement to one part of lime to five parts of sand by volume.

The panels were plastered on both sides with a 5/8-inch coat of gypsum plaster and a white finish coat.

The results of these measurements are as follows:

Frequency Band	Reduction Factor (in decibels)			
Cycles per sec.	Panel No. 1	Panel No. 2	Panel No. 3	Panel No. 4
118 to 138	30	31	30	33
241 to 271	30	31	35	33
486 to 538	38	36	44	42
977 to 1,071	48	47	52	46
1,956 to 2,140	53	50	56	49
3,961 to 4,231	59	58	65	62
Average	43	42	47	44

The reduction factors are given in decibels as the results are thereby expressed in numbers which are approximately proportional to the loudness of a sound as heard by the ear. The larger the reduction factor the better the sound insulation of the panel.

One decibel is practically equivalent to the smallest change in intensity that the average ear can detect. Because of the smallness of this unit a difference of less than 5 decibels is of no practical importance. It will be noticed that the average reduction factors for the panels measured differ by not over 5 decibels. The total reduction factor of say 45 decibels should be sufficient to reduce ordinary conversation to inaudibility.

The method of making these measurements is described at length in Bureau of Standards Scientific Paper S526. In brief, the test panel was built into an opening in one wall of a room specially constructed so that the other walls were as nearly sound proof as possible. The source of sound was a loud speaker located in this room and caused to rotate continually to break up the interference pattern. In addition, a frequency band was used in each case to give a more uniform distribution of the sound energy. The center points of the bands were at 128, 256, 512, 1,024, 2,048, and 4,096 cycles per second. The intensity of the sound was measured on each side of the wall, within and without the room in which the source of sound was located, and from the difference in intensity the foregoing results were calculated.

CONTROL OF RELATIVE HUMIDITY IN A SMALL INCLOSED SPACE.

In response to a demand for information about maintaining constant humidity in a small space, especially in a small cabinet in which some testing of paper can be done, an article has been published giving a brief description of suitable apparatus, together with a compilation of data for the preparation of solutions having the required relative vapor pressure. Tables are given containing data for sulphuric acid solutions, glycerine solutions, and saturated solutions of certain salts. In order to obtain a certain relative humidity one of the solutions is prepared having a relative vapor pressure corresponding to the desired relative humidity, and the air to be conditioned is washed with this solution or otherwise brought to vapor pressure equilibrium with it. Since the pressure of saturated water vapor changes rapidly with temperature change, resulting in a corresponding change in relative humidity for a given vapor pressure in the air, some form of temperature control is necessary. Several procedures are available, the choice depending upon the particular laboratory conditions and the accuracy required. The measurement of the relative humidity by means of a modified form of wet- and dry-bulb psychrometer is briefly described.

The article was published in *Paper Trade Journal*, Oct. 29, 1931, and reprints are available at the bureau.

DETERIORATION OF LEATHER.

Vegetable-tanned leather is often bleached with sulphuric acid in order to improve its color and appearance. This step in the process often occurs before the finishing or stuffing with oils and greases which form a part of nearly every commercial leather.

A study has been completed to determine whether the addition of grease to leather retarded or accelerated its deterioration by any sulphuric acid which might be present before the grease was applied. Leather was tanned with chestnut and quebracho extracts and treated so that samples containing 0, 1, 2, 3, and 4 per cent. sulphuric acid were obtained. A further division of the samples was made and

these were treated so as to contain 0, 10, and 20 per cent. by weight of a 50–50 mixture of cod oil and tallow.

All the samples were tested for tensile strength and then stored at 70° F. and 65 per cent. relative humidity. Selected samples were again tested for tensile strength after 6, 12, 18, and 24 months. The change in strength of the samples during aging was taken as a measure of their deterioration.

The results showed no significant difference in the rate and amount of deterioration occurred whether the leather contained grease or not. The addition of grease, therefore, does not appear to influence the action of the sulphuric acid which may be present in leather before the grease is applied.

Konel, A Platinum Substitute. (*Machinery*, December 1931.) In the early radio sets, tubes with platinum filaments were used. With the growth of the radio industry, as well as of other electrical applications of platinum, this metal was found to be rather too scarce and expensive for wide application. Through persistent research, a substitute was found in the form of a new alloy known as Konel, consisting mainly of nickel and cobalt. The new alloy, developed by the Westinghouse Laboratories, has proved itself not only equal, but superior to platinum for the purposes for which it has been developed.

Not only does it have the remarkable resistance to acids and corrosion possessed by platinum, but it has greater strength at red heat and better electrical qualities as a filament. There is one use for platinum, however, for which it cannot serve as a substitute, and that is for jewelry, for it costs not $100 an ounce or more, but only a few dollars a pound. One of the interesting characteristics of Konel is that it retains remarkable strength at high temperatures. In fact, if a steel wire of the same diameter as Konel wire is heated to a red heat, the hot Konel wire will be found to be stronger than the steel wire.

R.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, DECEMBER 16, 1931.

The regular monthly meeting of the Institute was called to order at eight-twenty by Dr. Howard McClenahan, Secretary, who presided over the meeting.

The Chairman announced that the minutes of the November meeting had been printed in full in the JOURNAL OF THE FRANKLIN INSTITUTE and unless there were objection, these minutes would be regarded as approved as printed. No objection was offered and they were declared approved.

The Secretary reported the following additions to the membership since the last report concerning such matters: four Resident members; six Non-Resident members; two Student members; three reinstatements.

The Secretary stated that the meeting was the occasion for the annual nomination of officers and members of the Board of Managers whose terms expire with this meeting and that the election has to take place at the regular January meeting of the Institute. He stated that the nominations must be made in writing and signed by at least two members, and must bear the statement that the nominee had agreed to serve if elected. He stated that he would read the list of nominations which were submitted by the Nominating Committee but would also receive any additional nominations from the floor.

He then presented the following report of the Nominating Committee:

Office	*Nominee*	*Nominated by*
President	Nathan Hayward	Henry Howson M. S. Morgan Elisha Lee
Vice-Presidents	Henry Howson C. C. Tutwiler Walton Forstall W. Chattin Wetherill	Clarence A. Hall James Barnes Arthur H. Thomas
Treasurer	Benjamin Franklin	Geo. A. Hoadley Charles E. Bonine
Board of Managers:		
To serve three years:	H. B. Allen G. H. Benzon, Jr. G. H. Clamer A. W. Goodspeed L. H. Kinnard Elisha Lee Charles Penrose G. D. Rosengarten	Haseltine Smith W. C. Wagner Morton Gibbons-Neff
To serve two years:	Charles H. Ewing Horace P. Liversidge	
To serve one year:	G. W. Pepper	

After presenting these the Chairman called for further nominations from the floor. No such nominations were offered and therefore the list of nominations

read by him was declared to be the list of official nominations and to constitute the ticket which would be presented in advance of the January meeting. The ballots will be distributed as always, by mail.

The Secretary called the attention of the membership to the Christmas Week Lectures of this year, which will be delivered by Dr. Dayton Clarence Miller, of the Case School of Applied Science, on "Science of Sound." He made a full statement concerning these lectures and urged the membership and friends of the Institute to regard tickets to the series as very appropriate Christmas gifts to young people.

The Secretary further stated that the list of lectures for the balance of the Institute year would be presented early in January, but read the list of subjects which would be covered in those lectures.

There was no further business so the Chairman introduced as the speaker of the evening Mr. A. P. H. Trivelli, in charge of Photomicrographic Research, Eastman Kodak Company, Rochester, New York. Mr. Trivelli read a delightful paper, fully illustrated by superb photomicrographs, made in the Eastman laboratories. The lecture was followed by an animated discussion which was much enjoyed, and the meeting was closed by an expression of appreciation and thanks to Mr. Trivelli for the trouble which he had given himself to come to Philadelphia to appear before the Institute, and for the quality of his lecture.

The meeting adjourned at nine-thirty-five p.m.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, December 9, 1931.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 9, 1931.

MR. LIONEL F. LEVY *in the Chair.*

The following reports were presented for final action:

No. 2930: Teleseismograph.

This report, recommending the award of the John Price Wetherill Medal to Doctor Frank Wenner, of the Bureau of Standards, Washington, D. C., "In consideration of his ingenious design of a recording teleseismograph of superior performance," was adopted.

No. 2941: Improvements in the Field of Absorption Refrigeration.

This report recommending the award of the John Price Wetherill Medal to Baltzer Carl von Platen and to Carl Georg Munters, of Stockholm, Sweden, "In consideration of their pioneer work and ingenuity in evolving principles effecting an absorption type of apparatus for refrigeration by using an inert gas to equalize the pressures required in the System."

The following report was presented for first reading:

The Franklin Medal.

GEO. A. HOADLEY,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, December 16, 1931.)

RESIDENT.

MR. FRANCIS B. FOLEY, Superintendent of Research, The Midvale Company. For Mailing: 3418 Warden Drive, Philadelphia, Pennsylvania.

DR. JULIUS NEUMUELLER, Optometrist, American Optical Company, 740 Sansom Street, Philadelphia, Pennsylvania. For Mailing: P. O. Box 182, Woodbury Heights, New Jersey.

DR. ARNO VIEHOEVER, Educator, Research Worker and Consultant Philadelphia College of Pharmacy and Science, Philadelphia, Pennsylvania. For Mailing: 210 Rutgers Avenue, Swarthmore, Pennsylvania.

NON RESIDENT.

MR. CARL BREER, Automotive Engineer, Chrysler Corporation, 341 Massachusetts Avenue, Detroit, Michigan.

MR. PHILIP JAMES KENT, Electrical Engineer, Engineering Department, Chrysler Corporation, Detroit, Michigan.

PROF. WALTER EDWIN LAY, Educator, 101 Engineering Annex, University of Michigan, Ann Arbor, Michigan.

MR. ROGER KENNETH LEE, Director of Research, Chrysler Motors Corporation. For Mailing: 190 Colorado Street, Detroit, Michigan.

MR. WILLIAM JOSEPH MURPHY, Construction Engineer, 52 Vesey Street, New York City. For Mailing: 74 Polk Street, West New York P. O., New Jersey.

MR. JAMES C. ZEDER, Assistant Chief Engineer, Chrysler Engineering Department, Chrysler Corporation, Detroit, Michigan.

STUDENT.

MR. HOWARD HOLBROOK SHEPPARD, Student, 224 West Haines Street, Philadelphia, Pennsylvania.

CHANGES OF ADDRESS.

MR. JOHN H. BARR, 505 East Seneca Street, Ithaca, New York.

MR. ONWARD BATES, 934 John's Road, Augusta, Georgia.

MR. J. BOEKEL, 509–19 Vine Street. Philadelphia, Pennsylvania.

DR. ROBERT H. BRADBURY, Colonial Village, Wayne, Pennsylvania.

MR. STERLING H. BUNNELL, Industrial Department, The National City Company, 55 Wall Street, New York City.

MR. FRANCIS L. CRAMP, 5120 Pulaski Avenue, Germantown, Philadelphia, Pennsylvania.

MR. CHESTER LESLIE DAVIS, Radio Corporation of America, 1227 National Press Building, Washington, D. C.

MR. DING U. DOO, Chiao-Fung University, Shanghai, China.

MR. HENRY C. EVANS, Box 191, Haverford, Pennsylvania.

DR. F. W. FRERICH, 4557 West Pine Boulevard, St. Louis, Missouri.

MR. F. LYNWOOD GARRISON, Westtown, Pennsylvania.

MR. ANTHONY A. HAUSSER, 117 Sylvania Street, Germantown, Philadelphia, Pennsylvania.

MR. E. S. HIGGINS, Parmelee System, Inc., 1775 Broadway, New York City.
MR. WILLIAM HOSKINS, 222 West Adams St., Room 325, Chicago, Illinois.
DR. GEORGE L. KELLEY, 15 Charlbury Road, Oxford, England.
MR. DONALD L. KELLOGG, Oil Well Supply Company, Oil City, Pennsylvania.
PROFESSOR J. C. MCLENNAN, 88 Prince Arthur Avenue, Toronto, Canada.
MR. A. J. VANDEN MEERSCHE, St. Pieters Aâlststraat, 20, Ghent, Belgium.
MR. RODOLPHO ORTENBLAD, c/o Pirelli S. A. Caixa Postal, 40, Rio de Janeiro, Brazil.
DR. GEORGE D. ROSENGARTEN, Royal Palm Dock, Miami, Florida.
MR. HERVEY J. SKINNER, President, Skinner and Sherman, Inc., 246 Stuart Street, Boston, Massachusetts.

NECROLOGY.

Mr. Rufus S. Sanford, Philadelphia, Pennsylvania.
Mr. Roberts le Boutillier, Wayne, Pennsylvania.

LIBRARY NOTES.

RECENT ADDITIONS.

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BOOK REVIEWS

THE PHYSICS OF HIGH PRESSURE. By P. W. Bridgman, Professor of Physics, Harvard University. vii + 398 pages, illustrations, tables, 8vo. New York, The Macmillan Company, 1931. Price $5.00.

Professor Bridgman has been investigating phenomena at high pressures continuously since 1906. Early in his work he developed a packing technique which makes it possible to reach without leak any pressure allowed by the mechanical strength of the containing vessels. With the aid of this technique he has extended his investigations over a wide range of pressures up to the highest ever attained in the laboratory.

The results of his studies have appeared in seventy-three monographs. In the aggregate these represent one of the most interesting and most successful projects undertaken in experimental science during the present century. With his own studies as a framework Professor Bridgman has now written a book which summarizes the present state of human knowledge in its field. Not more than half a dozen books covering this field have appeared previously.

The introductory chapters present an historical survey of experimentation at high pressure, an exposition of the technique of high pressure investigation, and of the measurement of high pressure. Then follow in succession chapters on rupture phenomena, on pressure-volume-temperature relations in fluids, on compressibility of solids, on melting phenomena, on polymorphic transition, on electrical resistance of metals and solids, on thermoelectric phenomena, on thermal conductivity, on viscosity, and on other effects of pressure.

At the end of each chapter appears a bibliography of outstanding publications in the field. Professor Bridgman's own papers are listed in an appendix in order of publication.

The text is illustrated with twenty-five tables of data, and eighty-seven figures showing graphical relations and diagrams of apparatus. Three full page plates show photographs of a typical high pressure installation and of rupture phenomena in metals.

The book is lucidly, even delightfully written. The reader soon finds himself the laboratory-guest of the author. Hobbies are displayed, successes and failures, hopes and plans are related, phenomena are exhibited, and their significance discussed.

The book is greeted as a long expected and welcome addition to the literature of physical science.

THOMAS D. COPE.

PROCEEDINGS OF THE THIRD INTERNATIONAL CONGRESS FOR APPLIED MECHANICS HELD IN STOCKHOLM FROM THE 24TH TO THE 29TH OF AUGUST, 1930. Three volumes, quarto, cloth. Volume I—Hydro- and Aerodynamics; xxii–458 pages, illustrations, plates; Volume II—Elasticity, Plasticity, Strength of Materials, Ballistics and Rational Mechanics, viii–474 pages, illustrations, plates; Volume III—Problems of Stability and Vibrations, viii–356 pages, illustrations, plates, published under the editorial supervision of C. W. Oseen and Woloddi Weibull. Stockholm, A. B. Sveriges Litografiska Tryckerier, 1931. Price 130 Kroner.

The term "applied mechanics" never seems to have achieved a well-defined meaning as a subject heading, and a hasty glance of the title of these volumes may not at once reveal that they include dissertations on topics ranging from those of interest mostly to the mathematical physicist to results directly useful to the engineer. There are in all over 180 papers on a wide variety of topics among whose contributors are recognized leaders of scientific thought in their respective fields in all parts of the world.

L. E. P.

CHEMICAL ENGINEERING CATALOG, 1931 (sixteenth annual) edition, collected, condensed and standardized catalog data of equipment, machinery, laboratory supplies, heavy and fine chemicals and raw materials used in the industries employing chemical processes of manufacture, with classified indexes of such equipment and materials, carefully cross-referenced, and a technical and scientific books section, cataloging and briefly describing a practically complete list of books in English on chemical and related subjects. 1017 pages, illustrations, quarto. New York, The Chemical Catalog Company, Inc., 1931.

This latest issue contains all the desirable features of the previous editions. There is also included a list of all firms making chemicals, whether they use space in the catalog or not. This will be a great convenience.

The trade name index, though not complete, lists over 1500 names of brands giving the name and address of the manufacturer in each case.

The technical and scientific books section gives data concerning recent works in all branches of applied science, principally in English, though some foreign publications are noticed.

A dozen pages are given over to announcements of the leading publishers and book dealers. As a reference work the Chemical Catalog should be useful.

R.

THE NEW "MUNRO AND JAMIESON," A POCKET-BOOK OF ELECTRICAL RULES AND TABLES FOR THE USE OF ELECTRICIANS AND ENGINEERS. Revised throughout under the Editorial Oversight of W. R. Cooper, A.M. Inst. C.E., M.I.E.E., and Rollo Appleyard, in association with eminent specialists, Twenty-third edition, viii–765 pages, 16mo. Cloth. Philadelphia, J. B. Lippincott Company, 1931. Price $4.00.

The "Munro and Jamieson" has enjoyed for many years a place of honor in the reference collection of the British electrical engineer, particularly in the field of telegraphy which early became a highly refined and successfully applied art in the hands of the scientists of that country. In the course of time the enormous growth of the industry has presented difficulties in keeping the volume within practical limits of size and particular attention has been given to what to displace of old matter and what to add of the recent advances. This twenty-third edition, the publishers state, is at a low price to meet a popular demand. The typographical quality, however, is excellent.

L. E. P.

SCIENCE IN ACTION. By Edward R. Weidlein and William A. Hamor, Mellon Institute of Industrial Research, Pittsburgh, Pa., 310 pages, ill., 8vo. New York, McGraw-Hill Book Company, Inc. Price $3.00.

Twin-authors Weidlein and Hamor have written exactly what they describe as a sketch of the value of scientific research in American Industries. A better word would be photograph. Although they have outlined with important strokes the evolution of research in industry, one is left with a photographic impression—provided, of course a sensitive plate.

The several parts of the book take up research as it bears on the necessities of life and (negative) happiness, medicine. I recall the remark of Michael Pupin that industries once so reluctant to use youth from the universities now are in danger of draining our academies of their best men. In fact they are forced to conspire together to maintain teaching staffs. Part III confirms this statement, made several years ago in the Franklin Inn.

The pilot function of research is to be taken very seriously. It is too obvious to deserve more than passing remark when one explores industrial waste, or the exploits of the plastics or the magic of chemical synthesis as one new industry after another has been born. The rationalization, with the indispensable help of research, of industrial management, of machines and their processes, of labor and of banking all have their place in the story. It is a grand story and would not be complete without an international aspect—at least in the twentieth century.

HOWARD W. ELKINTON.

THE ELEMENTS OF ECONOMIC GEOLOGY. By J. W. Gregory, D.Sc., F.R.S., M.I.M.M., Professor of Geology in the University of Glasgow, 312 pages, illustrations, 8vo. New York, E. P. Dutton & Co. Price $3.50.

Anyone who is engaged in the commerce of metals will find this book inter-

esting. It is written particularly for the benefit of those directly interested in mining and the discovery of ores.

After an introduction, which touches on the structure of the earth, one is directed promptly to the source of metals, the influence of waters and also to the intricacies of hydatogenesis, propylitization and pneumatolysis. Fortunately the gold-panner does not distress his mental equipment with an elaborate nomenclature, that is if he knows his ores.

Part II is devoted to detailed consideration of specific ores. Gold comes first in order of its importance and in the interest of an astute geologist, not to mention its importance to currency. The places where gold can be found are recorded for the especial use of the arm-chair gold-rush fan who inclines to follow, in a book, the sweat and tears of the dirt miner. No one can afford to be untutored on platinum particularly if choosing a ring in the modern manner. Tin and tungsten bring one down to earth, the former with a most romantic past in the Cornish mines first worked by the Phœnicians, the latter with a utilitarian present in the electric lamp overhead. Copper (the red metal that tips the horns of the bulls and sheathes the claws of the bears), lead, zinc and silver all enjoy space. The five minor metals of nickel, mercury, antimony, arsenic and bismuth deserve the passing mention they receive, but the ores of manganese, chromium, aluminum and iron require an affectionate regard.

Please do not make a mistake, Professor Gregory nowhere yields to romance or to any evidence of sentiment. He writes with the approved British restraint. One is lucky if one stumbles on the lode of hidden humor. As usual, the close print, meager black ink drawings conform completely with the English demand for a dull format. However, a book for miners justifies a little digging.

The earthy minerals are covered in Part III, micas, asbestos and gems (thrown in for sparkle), clay, building stone and road metals, cements, the soil, nitrates and phosphates and, of tremendous economic importance, salt. Geology cannot and should not confine itself to the minerals that bed the earth. Soap is prepared and sold to meet the waters of a district. Every city engineer must treat with the hardness and softness of water. It is immensely important whether drinking water drains a slum or is released from the magma of the earth's crust. Nor can geology disregard the sea and least of all the earthquake, the cause, the forecast and the reception of which is hardly yet within the threshold of our knowledge.

The last two chapters on coal and fuel oil, in themselves, well repay the persistence of the reader, adult or minor.

HOWARD W. ELKINTON.

NUCLEIC ACIDS. By P. A. Levene, The Rockefeller Institute for Medical Research, and Lawrence W. Bass, Mellon Institute of Industrial Research, now Assistant Director of Research, the Borden Company. American Chemical Society Monograph Series, 337 pages, tables, 8vo. New York, The Chemical Catalog Co., Inc. Price $4.50.

One can do no better than to quote from the authors' Preface: "The chemistry of nucleic acids can be summed up very briefly. Indeed, a few graphic formulas, which need not fill even a single printed page might suffice to express the entire store of our present-day knowledge on the subject. Yet a detailed formula

expressing the arrangement of all the atoms entering in the structure of the molecule of nucleic acids is the result of the combined labors of many chemists, the list of whom is headed by Scheele, who was born in 1742 and died in 1786. Somehow, every point in the structure of nucleic acids was reached with great difficulty by the paths of error and controversy. Theories had been advanced and had been abandoned but even the errors often led to true progress and it would be an injustice to many if the monograph contained only the views that seem correct today. Hence, the historical method of discussion was adopted in this monograph. It was hoped that this method would lead to an unbiased presentation of the material; yet we fear that unintentionally some contributions may have been overlooked."

The senior author, P. A. Levene, a member of the Rockefeller Institute for Medical Research, has been closely associated with the problems of biological chemistry for many years and probably there is no one better fitted than he to direct the preparation of this monograph on a subject which has engaged his active attention for a number of years. To a person not directly concerned with a close study of nucleic acids the Introduction will prove to be one of the most engaging chapters of the book. The authors recall the days contemporary with the discovery of the cell as the structural unit of animate matter. With this discovery biology began a period of rapidly accelerating development which with the aid of physics and chemistry appears to stand today on the brink of disclosures more startling and significant than ever.

Armed with this knowledge concerning the nucleic acids, biological chemists have united to make a concerted attack upon the tuberculosis bacilli and hope, that by determining among other things, the types of nucleic acid found in this organism, they will have uncovered some of the vulnerable spots in its formerly supposed armor of immunity. Although all the available knowledge may be summarized on one or two pages, volumes have been written during the tedious process of this step by step advance. In the monograph, "Nucleic Acids," the authors carefully follow this development from its beginnings to the present. Such mode of treatment is essential to those who would know biological chemistry. The successes and failures of former workers must provide the tools for the future experimentalist and man of research.

The book is divided in two parts. Part One deals with the "Components of the Nucleic Acids." These consist of sugars, imidines, pyrimidines, uric acid, purine, purine bases, nucleosides and nucleotides. This section comprises the greater portion of the book and is relatively complete in character. Various methods are given for preparing and synthesizing these components. In addition, their historical backgrounds are briefly recounted; the working out of the structural formulas, described and their properties summarized.

Part Two deals with the nucleic acids themselves. It is rather interesting to contemplate that when phosphoric acid, a pentose sugar and a purine or pyrimidine are combined in a certain way a nucleic acid results. That this acid is a very essential part of the cell nucleus and in the nucleus is a substance known as chromatin which has much to do with determining the character of the living organism.

In this second part there is a chapter on the discovery of nucleic acids followed by one more extensive dealing with their structure. Establishment of

their structures has been a difficult problem and only in recent years have sufficient corroborating facts been assembled to leave little doubt in the minds of most workers as to what the true structural constitution may be. Nucleic acids of higher order are accorded a separate chapter. Work on these is still in the early stages and comparatively little is known about them.

Nuclease is a term applied to enzymes which have the ability to disrupt nucleic acids. They have been divided into three classes depending upon specific action. Nucleinase breaks down the nucleic acid into nucleotides—single phosphoric acid, sugar, purine or pyrimidine combinations. Nucleotidase liberates phosphoric acid from the nucleotide. Nucleosidase breaks the bond between the sugar and the purine or pyrimidine base.

T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 396. Hydraulics of Fuel Injection Pumps for Compression-Ignition Engines, by A. M. Rothrock, 47 pages, illustrations, quarto. Washington, Superintendent of Documents, 1931. Price twenty-five cents.

Formulas are derived for computing the instantaneous pressures delivered by a fuel pump. The first derivation considers the compressibility of the fuel and the second, the compressibility, elasticity, and inertia of the fuel. The second derivation follows that given by Sass; it is shown to be the more accurate of the two. Additional formulas are given for determining the resistance losses in the injection tube. Experimental data are presented in support of the analyses. The report is concluded with an application of the theory to the design of fuel pump injection systems for which sample calculations are included.

Report No. 398. Investigation of Damping Liquids for Aircraft Instruments —II, by M. R. Houseman and G. H. Keulegan, 19 pages, illustrations, quarto. Washington, Superintendent of Documents, 1931. Price fifteen cents.

Data are presented on the kinematic viscosity, in the temperature range $-50°$ to $+30°$ C., of pure liquids and of solutions of animal oils, vegetable oils, mineral oils, glycerine, and ethylene glycol in various low freezing point solvents. It is shown that the thermal coefficient of kinematic viscosity as a function of the kinematic viscosity of the solutions of glycerine and ethylene glycol in alcohols is practically independent of the temperature and the chemical composition of the individual liquids. This is similarly true for the mineral oil group and, for a limited temperature interval, for the pure animal and vegetable oils.

The efficiency of β-naphthol, hydroquinone, and diphenylamine to inhibit the change of viscosity of poppy-seed and linseed oils was also investigated.

Report No. 400. The Aerodynamic Characteristics of a Slotted Clark Y Wing as Affected by the Auxiliary Airfoil Position, by Carl J. Wenzinger and Joseph A. Shortal, 16 pages, illustrations, quarto. Washington, Superintendent of Documents, 1931. Price fifteen cents.

Aerodynamic force tests on a slotted Clark Y wing were conducted in the vertical wind tunnel of the National Advisory Committee for Aeronautics to determine the best position for a given auxiliary airfoil with respect to the main

wing. A systematic series of 100 changes in location of the auxiliary airfoil were made to cover all the probable useful ranges of slot gap, slot width, and slot depth. The results of the investigation may be applied to the design of automatic or controlled slots on wings with geometric characteristics similar to the wing tested.

An increase of 41.5 per cent. in the maximum lift above that of the plain wing was obtained for the slotted Clark Y wing. At the same time, the angle of attack for maximum lift was increased 13°. It was found that a maximum increase of about 30° was possible in the highest stalling angle, but at a maximum lift coefficient slightly less than that of the plain wing. However, with one slot position, an increase of 25°, together with an increase in the maximum lift coefficient of 23.3 per cent., was obtained. The best positions of the auxiliary airfoil were covered by the range of the tests, and the position for desired aerodynamic characteristics may easily be obtained from charts prepared especially for the purpose.

R.

PUBLICATIONS RECEIVED.

Colloid Chemistry, Theoretical and Applied, by Selected International Contributors, collected and edited by Jerome Alexander, volume 3. 655 pages, tables, diagrams, 8vo. New York, The Chemical Catalog Company, Inc., 1931. Price $10.50.

Cours de l'Ecole Polytechnique, Cours de Physique, par Ch. Fabry, Tome I, 663 pages, quarto. Paris, Gauthier-Villars et Cie, 1932.

Signals from the Stars, by George Ellery Hale, 138 pages, plates, illustrations, 12mo. New York, Charles Scribner's Sons, 1931. Price $2.00.

Bibliotheque des Annales des Postes, Telegraphes et Telephones, Cours d'Electricite Theorique, par J.-B. Pomey, Tome III, 312 pages, 8vo. Paris, Gauthier-Villars et Cie, 1931.

Abridged Scientific Publications from the Kodak Research Laboratories, Volume 14, 1930, 295 pages, tables, illustrations, 8vo. Rochester, Eastman Kodak Company, 1931.

Brazil—Ministerio da Agricultura, Diretoria de Meteorologia. O Segundo Ano Polar Internacional, 1932–1933, a Cooperacao do Brasil, por Sampaio Ferraz. 16 pages, 8vo. Rio de Janeiro, Ministerio da Agritultura, 1931.

Ontario Department of Mines, Fortieth Annual Report, Vol. 40, Part 2, 1931, 43 pages, illustrations, tables, maps, 8vo. Toronto, King's Printer, 1931.

United States Department of Agriculture, Miscellaneous Publication No. 118, The Influence of Weather on Crops: 1900–1930. 245 pages, 8vo. Washington, Government Printing Office, 1931. Price forty cents. Technical Bulletin No. 265, Electrical Equipment on Movable Bridges, by Conde B. McCullough, Albin L. Gemeny, W. R. Wickerham, 111 pages, diagrams, illustrations, 8vo. Washington, Government Printing Office, 1931. Price thirty-five cents. Report of the Chief of the Food and Drug Administration, 30 pages, tables, 8vo. Washington, Government Printing Office, 1931. Report of the Chief of the Bureau of Chemistry and Soils, 78 pages, tables, 8vo. Washington, Government Printing Office, 1931. Report of the Chief of the Bureau of Plant Industry, 38 pages, 8vo. Washington, Government Printing Office, 1931. Report of the Chief of the Bureau of Biological Survey, 63 pages, tables, 8vo. Washington, Government Printing Office, 1931.

U. S. Bureau of Mines, Mineral Resources of the United States, 1930 (Summary) 120 pages, 8vo. Washington, Government Printing Office, 1931. Price twenty cents. Annual Report of the Director of the Bureau of Mines to the Secretary of Commerce, for the Fiscal Year Ended June 30, 1931. 61 pages, diagrams, tables, 8vo. Washington, Government Printing Office, 1931. Price ten cents. Bulletin 343, Permissible Coal-Handling Equipment, 91 pages, illustrations, 8vo. Washington, Government Printing Office, 1931. Price thirty-five cents. Bulletin 347, Gases that Occur in Metal Mines, 21 pages, 8vo. Washington, Government Printing Office, 1931. Price five cents. Technical Paper 505, Influence of Fractionation on Distribution of Sulphur in Gasoline, by Ralph H. Espach and H. P. Rue, 24 pages, tables, plates, 8vo. Washington, Government Printing Office, 1931. Price ten cents.

National Advisory Committee for Aeronautics. Technical Notes, No. 396, Performance of a Compression-Ignition Engine with a Precombustion Chamber Having High-Velocity Air Flow, by J. A. Spanogle and C. S. Moore, Langley Memorial Aeronautical Laboratory, 15 pages, diagrams, quarto. Washington, Committee, 1931. No. 399, Some Characteristics of Fuel Sprays at Low-Injection Pressures, by A. M. Rothrock and C. D. Waldron, 5 pages, illustrations, 8vo. Washington Committee, 1931. No. 400, Advantages of Oxide Films as Bases for Aluminum Pigmented Surface Coatings for Aluminum Alloys, by R. W. Buzzard and W. H. Mutchler, Bureau of Standards, 16 pages, illustrations, quarto. Washington, Committee, 1931. No. 401, Tests of N. A. C. A. Airfoils in the Variable Density Wind Tunnel. Series 44 and 64, by Eastman N. Jacobs and Robert M. Pinkerton, Langley Memorial Aeronautical Laboratory, 31 pages, illustrations, quarto. Washington, Committee, 1931.

CURRENT TOPICS.

Ferromagnetism and Hysteresis. SIR ALFRED EWING. (*Proc. Phys. Soc.*, London, Aug. 15, 1930.) On May 23, 1930, the Physical Society held a discussion on the subject of magnetism. The reports of the papers presented and of the subsequent discussions occupy more than a hundred pages in the Proceedings and it would be difficult to indicate a better source of information on modern views of magnetism than this collection affords. Prof. B. W. Holman, however, expressed his disappointment over the disregard by the participants of the magnetic discoveries of Baron von Reichenbach, the mineralogist, and of W. M. Mordey. The early investigations of the former were published in Liebig's Annalen in 1845 and were translated into English in 1850. Mr. Holman has read von Reichenbach's work and adds that "It might interest some members to know that a worker at Faraday House has systematically investigated the extraordinary phenomena recorded by von Reichenbach—in particular the rendering of galena and quartz responsive to an ordinary permanent magnet. I have been shown a piece of transparent mineral which was apparently quartz (I was not allowed to touch it), and responded to a permanent magnet and had pronounced phosphorescence. The phosphorescence was so strong that it could be observed readily when the piece of alleged quartz was placed on the shelf of a cupboard in the laboratory and the doors partially closed." Still further regret was uttered that no attention had been given to the experiments of W. M. Mordey's on the behavior of paramagnetic substances in two-phase alternating fields and to his work on separating minerals by magnetic methods. It seems appropriate that the first paper should come from the dean of English magneticians, Sir Alfred Ewing, formerly professor in the University of Cambridge, whose book "Magnetic Induction in Iron and Other Metals" was so modern and so authoritative forty years ago. He began thus: "As a hopelessly old-fashioned magnetician, I find it an embarrassing compliment to be asked to open this discussion. You are taking down a dusty piece of apparatus from the upper shelf on which it was long ago put, and inviting it to function. It can do so only in pretty much the old way. I have nothing new to offer. Perhaps the best service I can render is to recall some aspects of the subject which were problems when I began to study magnetism fifty years ago, and are problems still."

His work, he says, had to do exclusively with ferromagnetics, iron, cobalt, nickel and their alloys. He emphasizes "the gulf that

divides ferromagnetics from other substances. It is a difference in kind, not simply in degree. Of the distinguishing features none is more significant than hysteresis. In my own early experiments it forced itself on my attention at every turn. I became soaked in hysteresis, and was led to invent the name." Since hysteresis is found only in ferromagnetic solids it seems to be connected with the retentivity of such substances.

Magnetic saturation can be thought of as due to the orienting of Weber's elementary magnets. These cannot be atoms because each atom is fixed in the space-lattice and the turning of atoms should affect the cohesion of the material, but magnetizing a ferromagnetic does not change this property. "I have tried the experiment of strongly magnetizing a piece of iron in a testing machine, while it carried a load which very nearly made it yield, and have found that repeated reversals of magnetism did nothing to reduce the elastic resistance. We must rather think of something within each atom that can turn in response to an applied magnetic force, something which permanently possesses a magnetic moment sufficient to provide the saturation value when in every atom it is brought into parallelism with the applied force."

He speaks of his at one time familiar model of a ferromagnetic consisting of an aggregate of compass needles equally spaced. Their poles mutually attracted and repelled one another and when they were placed in a coil of wire traversed by a slowly alternating current they, by their shifts of position, produced a total magnetic effect that in its changes closely simulated the sequence of magnetic relations caused in a piece of iron placed in the axis of an electromagnet energized by the same kind of current. The speaker told that Swinburne objected to his idea of the control of the positions of the magnetic elements in a ferromagnetic by their actions on one another and, to show its impossibility, deduced by theory a consequence that at that time appeared improbable. Six years later, however, F. G. Baily found by experiment that the supposedly absurd consequence actually agreed with the facts.

While the speaker recognizes that "models have gone out of fashion now" he recommends that students of magnetism at least give more attention to ferromagnetics, because there magnetism is seen on the largest scale and because he believes that there the most significant clues to the field of magnetism are to be found.

G. F. S.

Infrared Filters of Controllable Transmission. A. H. PFUND. (*Phys. Rev.*, Vol. 36, No. 1.) The new type of optical filter here described was devised for the specific purpose of making more

available the large dispersion and high resolving power of the diffraction echelette grating. Its drawback lies in the superposition of spectra of different orders. To the same place are directed simultaneously wave-lengths in the ratios of 1, $\frac{1}{2}$, $\frac{1}{3}$, etc. The problem is how to cut out the extraneous wave-lengths and leave only the longest. "Methods now in use for filtering out the undesirable radiations are here enumerated: I. Infrared monochromator. II. Residual rays. III. Reflection from rough surfaces. IV. True absorption filters. Most of them are limited in the range of spectrum which can be covered; again some lack flexibility, i.e. if true filters or crystals (employed for residual rays) fail to show high transmission or reflection, respectively, in the region to be studied. There is nothing to be done about it." Long before the close of the nineteenth century the then Lord Rayleigh showed that the amount of light scattered by particles small in size in comparison with the wave-length of the light varies inversely as the fourth power of the wave-length. "If, however, the particle size be large in comparison with the wave-length, the particles assume the rôle of small mirrors which reflect vigorously. Consequently, if transparent particles of a uniform and suitable size be produced it will be found that they reflect the regions of short wave-lengths strongly and scatter those of great wave-length feebly. Used in transparent light a film of such small particles will be virtually opaque to the short wave-lengths and highly transparent to the longer. Body color, involving true absorption, plays no necessary part in the functioning of these filters."

The material which, in the form of finely divided particles, composes the filter must be transparent to the type of radiation used. For wave-lengths shorter than 20 μ (μ is .001 mm.) oxides, sulphides and halides are most transparent. Pfund finds the most satisfactory films to be made by catching the oxide fumes from burning metal on a plate. Further, the particles should be uniform in size and should have a considerable range of dimensions. "Air-flotation" or successive centrifuging will meet these conditions. To avoid any effect of the transparency of the plate supporting the particles, they are deposited on a polished speculum surface. This mirror treats all wave-lengths alike by reflecting them regularly, while the film will reflect only the short waves diffusely, letting the long waves go through to be reflected by the mirror and to traverse the film a second time on their way out. An objection to this reflection type of filter is found in the admixture of diffused light of short wave-length with the desired longer wave-length.

From the transmission curves for ZnO and MgO it appears that the former transmits about 80 per cent. of radiation having the

wave-length of 2.5 μ, while MgO transmits only 10 to 20 per cent. To get a transmission of 80 per cent with MgO the wave-length must be close to 5 μ. These differences agree with the fact that the zinc oxide particles are smaller than those of magnesium oxide. With both oxides the percentage of transmission increases with the wave-length of the incident radiation.

The total energy associated with each wave-length cannot be judged by the percentage of transmission alone. It is the continued product of these four quantities, (*a*) energy leaving the source, (*b*) transmission of entire path in air, (*c*) reflection factor for grating, and (*d*) transmission factor for filter. The relative energies for the range of wave-lengths emitted by a Nernst lamp after they have fallen on films of four materials are given. For ZnO the maximum energy is at 2.1 μ, for MgO at 3.3 μ, for white lead spread on the mirror as a paste, 5 μ and for silica similarly applied 7.4 μ. The average diameter of the four kinds of particles are respectively .22, .35, .50 and 2.5 μ. "Inspection shows that a very fair degree of separation between short and long waves has been effected and that, each filter may be used to advantage in the region of maximum energy."

G. F. S.

The Raman Effect. Its Significance for Physics and Chemistry. S. BHAGAVANTAM. (*Indian Journ. Physics*, Vol. V, Part III.) At the meetings commemorating the centenary of The Franklin Institute in 1924 among the foreign guests was Chandrasekhar Venkatarain Raman, professor of physics in the University of Calcutta. He was at that time thirty-six years of age. A Tamil by birth he had become an examiner in political economy for the British government. By sheer force of native ability he had carried his self-directed studies of physics and mathematics to so advanced a position that, deserting economics, he had become a Fellow of the Royal Society on account of his researches in physics. He had been in attendance on the British Association for the Advancement of Science in 1924 at Toronto and after its adjournment had accompanied the excursion to the Pacific coast. In some way it came about, perhaps by the timely arrival of a birthday, that one of the ladies of the party was in receipt of poetic homage from her friends. The contribution of Prof. Raman was a poem in Sanskrit addressed to her. In 1929 after his return to India after having spent some time on the staff of the California Institute of Technology he was knighted by George V and in 1930 he received the Nobel Prize in Physics.

"In an essay published by the Calcutta University in February, 1922, Prof. Raman outlined his programme of research which included a comprehensive study of the scattering of light in numerous substances over the widest range of physical states and conditions, and a comparison of the results obtained with the consequences of existing theories; it was confidently anticipated that the results would throw light on fundamental questions concerning the structure of molecules, molecular aggregation in solids and liquids, and upon the nature of radiation itself. The programme thus outlined was systematically pursued, and nearly 100 communications emerged from the laboratory during the six years following the publication of the essay. They dealt with the subject in all its varied aspects and relations to other fields of research." In 1928 Professor Raman discussed "A New Radiation" that had come to be recognized as a product of these researches. The new optical phenomenon now known as the "Raman Effect" is thus described by Raman and Krishnan "When monochromatic light is scattered in a transparent medium (be it gas, vapor, liquid, amorphous solid or crystal), the diffused radiation ceases to be monochromatic, and several new lines or sometimes bands (associated in many cases with a continuous spectrum) appear in the spectrograms of the diffused radiation. Further, the new radiations are, in general, strongly polarised. Our preliminary studies have proved conclusively that the effect arises in the following way: The incident quantum of radiation is either scattered as a whole, in which case we have the classical scattering, or else is absorbed in part by the molecules of the medium, the remaining part appearing as a scattered quantum. The part absorbed shifts the molecule to a level of energy different from the initial state. The possibility of a process of this kind, in respect of the electronic state of an atom, was first contemplated by Smekal, and figures prominently in the theory of dispersion due to Kremers and Heisenberg, and in the papers of Schroedinger. Our experiments furnish definite proof of the possibility of such processes, and show that they may occur also in such complicated systems as the molecules of a vapor or a liquid or even in a complete crystal." To take a concrete illustration, let the monochromatic radiation corresponding to a single line emitted by a strong mercury arc pass through dust-free benzene and by an exposure of some 40 hours, as was at first necessary, let a spectrogram of the light scattered at right angles to the beam be formed. In addition to the strong line due to the incident radiation there appear 8 lines with wave-lengths longer and 3 with wave-lengths shorter than this line. For each of these lines let the wave-number, i.e. the number of wave-lengths occupying one cm. of length, be calculated. The 8 waves have

smaller and the 3 waves have greater wave numbers than that of the original radiation. Now form the difference between the wave number of the original radiation and each of the others. Most of the differences coming from the group of 8 lines are found to agree with the wave numbers of lines due to the benzene molecule in the infra-red. This seems to come about in the following way, the quantity of energy in a quantum of radiation of any wave-length equals the wave number multiplied by a constant. A quantum of the incident radiation falling on a molecule of benzene sets up within it a vibration that requires and abstracts from the incident quantum a quantum of smaller value and the energy that equals the difference between these two quanta travels away as a quantum necessarily smaller than the original one and thus corresponding to a larger wave-length or to a smaller wave number.

The lines shorter in wave-length than the original line have a different explanation. Let the incident quantum impinge on a molecule already in an excited state. The molecule may return to its normal state emitting energy that adds itself to the original quantum thereby forming a quantum of greater value than the first and corresponding to larger wave number or a shorter wave-length. There is furthermore a case in which the difference between two wave numbers both due to molecular vibration manifests itself in the series of differences. "Each line of the incident spectrum, if of sufficient intensity, gives rise to its own set of lines or bands and associated continuous spectrum; the frequency shifts are independent of the incident radiation and are therefore characteristic of the particular material studied."

The Raman lines vary greatly in their intensity and width and are often strongly polarized. "The continuous spectrum also shows great variations in intensity with different materials."

The Raman effect makes it possible to use photography in studying molecular spectra in the infra-red. It is a great gain to be thus enabled to measure wave-lengths greater than .015 mm. Furthermore the information obtained is more definite than that coming directly from the infra-red spectrum. In the case of crystals X-ray diffraction methods provide information concerning the electrons and atoms effective in the scattering of X-rays. The Raman effect furnishes "just the kind of information that the X-rays do not give, concerning the strength and nature of the forces that hold the crystal together." For the last forty years the dissociation of electrolytes in solution has been much discussed. "The Raman effect offers an ideal method for investigating this problem, as spectra can be obtained both for the pure substance and also for its aqueous solutions at all possible concentrations. The

observed position and intensities of the lines enable us directly to determine the nature and number of the ions produced and thus to decide the question whether the dissociation is complete or only partial in strong solutions." Problems of molecular association and polemerisation also yield to the application of the Raman effect.

When the spectrum of light diffused by rock-salt is examined there are no Raman lines corresponding to the rest-strahlen of that substance. The Raman lines for the SO_4 group are strong in the case of diamagnetic crystalline sulphates such as those of magnesium and of zinc but they are weak or altogether lacking for paramagnetic sulphates, such as those of iron, nickel and copper. In organic compounds the Raman lines are strong and afford a method of studying chemical constitution. Enough has been said to show how far reaching are the applications of Raman's discovery.

The author gives a bibliography extending to July 1, 1930. There are 26 headings and about 350 references are included. An index of authors who have written on the Raman effect embraces more than 200 names, while a substance index occupies four and a half pages.

G. F. S.

Dust Studies. Studies by the United States Public Health Service have shown that there is a great difference in the harmfulness of dust. Of those studied, the only dust which had a really serious or fatal effect upon the worker was that containing free silica. In the investigation of granite cutting plants, most of the workers were found to be exposed to an average of about 60 million particles of dust per cubic foot of air. The dust contained about 70 per cent. silica, of which about 35 per cent. was in the form of quartz or free silica. Under such conditions there was an almost universal occurrence of silicosis, and a large proportion of the workers developed pulmonary tuberculosis. This disease, as a rule, did not make its appearance until after some 20 years of exposure, but when it did develop, a fatal result within less than two years was almost certain.

It used to be thought that the harmful effect of silica dust lay in the sharpness of particles, but now it is believed that some chemical effect is present, associated with the slow dissolving of the free silica particles.

It was not found in this investigation that it would be necessary to eliminate all of the dust to do away with the hazard. A safe limit of somewhere between 9 and 20 million particles of dust per cubic foot of air could be established.

The highest concentrations of dust were found in coal mining, both hard and soft. In these studies there was no great excess of sickness, but there did appear to be a slowly developing fibrosis which was in a way parallel with the early silicosis found among persons exposed to silica dust, but without the tendency toward tuberculosis.

A large amount of dust was also found in a cement plant. This plant was not typical, but was chosen because a great deal of calcium dust was present. Although no permanently serious effects appeared in this investigation, the workers were subject to a higher frequency of upper minor respiratory diseases and some other conditions.

The other studies have added considerable information of a negative nature. In the concentrations found, dust encountered in silver polishing, in a cotton plant, and that to which street-sweepers were exposed, appeared to have no harmful effect upon the workers. In all of these cases the concentrations were relatively low. In the case of street sweepers, however, the air breathed had a higher concentration than that to which the average city dweller is exposed.

The studies in question were not designed to cover all of the dusty trades in this country, but rather to consider specific types of dust. It is known that silicosis and tuberculosis are a serious hazard in certain mining operations, and this question is under continual investigation by the United States Bureau of Mines. The pottery industry, subway construction in granite rock, and many other occupations have a severe silicosis hazard.

The mitigation of the dust hazard in industry is primarily a ventilation problem. In many industries steps have been taken to prevent the escape of dust into the room by surrounding the machine with a suction hood or other device that removes the dust as it forms. This removal of dust at its source is the most important step to be taken.

The important points are the seriousness of the dust hazard, the large number of workers exposed, the varying effect of different dusts, the fact that a certain amount of dust has to be present before a hazard exists, and the necessity for removal of the dust at its source.

R.

Radio and Drought. (U. S. Department of Agriculture, Clip Sheet No. 703.) Radio broadcasting, however much it may be affected by atmospheric conditions, in no way influences the weather, says Dr. W. J. Humphreys, of the United States Weather Bureau. Therefore, he points out, suppression of the radio, many appeals

for which are received by Weather Bureau officials, on the score of its adverse effect on the climate, would prevent neither droughts nor floods.

Some of the pleas for abolishing the radio are based on the supposition that wireless broadcasting burns up the water vapor of the air, thus causing disastrous droughts. Others apparently are inspired by a belief that radio gives rise to excessive precipitation, which ends in a flood.

The fallacy of both these ideas is apparent from a careful analysis of the way nature makes rain. Rain, Doctor Humphreys explains, is caused by (1) evaporation of water, which depends on the temperature of the evaporating water, on the area of the evaporating surface, on wind velocity, and on the dryness of the air, and (2) condensation of water vapor, which depends on the presence of excessively small particles of sea salt, land dust, or other substances that take up water vapor, and on an adequate cooling of the vapor.

Obviously, Doctor Humphreys observes, radio can have no effect on any of these factors that enter into evaporation of water from the earth's surface or into the condensation of atmospheric vapor. Attributing droughts and floods to radio, in his opinion, is just another instance of the common human habit of ascribing a little understood occurrence to something that is even more mysterious.

R.

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